

Markscheme

November 2017

Mathematical studies

Standard level

Paper 1

25 pages



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Paper 1 Markscheme Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6.

1 Abbreviations

The markscheme may make use of the following abbreviations:

- M Marks awarded for Method
- A Marks awarded for an **Answer** or for **Accuracy**
- **C** Marks awarded for **Correct** answers (irrespective of working shown)
- **R** Marks awarded for clear **Reasoning**
- ft Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

- (a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If the candidate has full marks on a question use the C6 annotation, if the candidate has made an attempt but scores zero marks use C0. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations MUST be shown.
- (c) In this paper, if the correct answer is seen on the answer line the maximum mark is awarded. There is no need to check the working! Award C marks and move on.
- (d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.
- (e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.
- (f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.
- (g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.
- (h) If correct working results in a correct answer in the working box but then further working is developed, indicating a lack of mathematical understanding full marks should not be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer as long as it is unambiguous. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 (incorrect decimal value)	Award the final (A1) (ignore the further working)
2.	(x-6)(x+1)	x = 6 and -1	Do not award the final (A1) (see next example)

Example: Factorise $x^2 - 5x - 6$

Markscl	heme		Candidates' Scripts	Marking
(x-6)(x+1)	(A1)(A1)	(i)	Answer line: $(x+6)(x+1)$	(A0)(A1)
		(ii)	Working box: $(x-6)(x+1)$ followed by $x=6$ and -1 , or just $6, -1$	(A1)
			in either working box or on answer line.	(A0)

3 Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through** (ft) marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with "(ft)'.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

	Markscheme	Candidates' Scripts		Marking
(a)	$\frac{\sin A}{3} = \frac{\sin 30}{4}$ (M1)(A1)	(a)	$\frac{\sin A}{4} = \frac{\sin 30}{3}$	(M1)(A0)
				(use of sine rule but with wrong values)
	$A = 22.0^{\circ} (22.0243)$ (A1)			(A0) here was not marked (ft) and because there was an earlier estion part.)
(b)	$x = 7 \tan (22.0243^{\circ})$ (M1)	(b)	case (i) $x = 7 \tan 41.8^{\circ}$	(M1)
	= 2.83 (2.83163) (A1)(ft)	but	= 6.26 case (ii) 6.26	(A1)(ft) (C0) since no working shown

4 Using the Markscheme

- (a) A marks are dependent on the preceding M mark being awarded, it is not possible to award (MO)(A1). Once an (MO) has been awarded, all subsequent A marks are lost in that part of the question, even if calculations are performed correctly, until the next M mark. The only exception will be for an answer where the accuracy is specified in the question – see section 5.
- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** is not awarded for a correct answer if no reason or the wrong reason is given.
- (c) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(d) Unless the question specifies otherwise, accept equivalent forms. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value $\left(\text{for example } \frac{2}{3} \text{ if applicable} \right);$

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(e) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1.7; 1,7.

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49.

Different descriptions of an interval: 3 < x < 5; (3, 5); 3, 5 [.

Different forms of notation for set properties (e.g. complement): A'; \overline{A} ; A^c ; U - A; $(A; U \setminus A)$.

Different forms of logic notation: $\neg p$; p'; \tilde{p} ; $\neg p$.

$$p \Rightarrow q; p \rightarrow q; q \leftarrow p.$$

Significance level may be written as α .

(f) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader. As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award *(A1)* and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

- 2. If the candidate's unrounded answer is **not** seen then award (A1) if the answer given is **correctly** rounded to 2 or more significant figures, otherwise (A0).
- **Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
- **3.** If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

		If candidates final answer is given								
	Exact or to 4 or more sf (and would round to the correct 3 sf)	Correct to 3 sf	Incorrect to 3 sf	Correct to 2 sf ³	Incorrect to 2 sf	Correct or incorrect to 1 sf				
Unrounded answer seen ¹	Award the	Award the final (A1) irrespective of correct or incorrect rounding								
Unrounded answer not seen ²	(A1)	(A1)	(A0)	(A1)	(A0)	(A0)				
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen.3								

Examples:

Markschem	е		Candidates' Scripts		Marking
9.43 (9.43398)	(A1)	(i)	9.43398 is seen in the working followed by 9; 9.4; 9.43; 9.434 (correctly rounded)		(A1)
		(ii)	9.43398 is seen in the working followed by 9.433; 9.44 <i>etc</i> . (incorrerounded)		(A1)
		(iii)	9.4		(A1)
		(iv)	9		(A0) (correct to 1 sf)
		(v)	9.3	(incorrectly	(A0) rounded to 2 sf)
		(vi)	9.44	(incorrectly	(A0) rounded to 3 sf)

Markschem	е		Candidates' Scripts		Marking
7.44 (7.43798)	(A1)	(i)	7.43798 is seen in the working bo followed by 7; 7.4; 7.44; 7.438 et (correctly rounded)		(A1)
		(ii)	7.43798 is seen in the working bo followed by 7.437; 7.43 <i>etc.</i> (incorrect rounded)		(A1)
		(iii)	7.4		(A1)
		(iv)	7		(A0) (correct to 1 sf)
		(v)	7.5 (inc	correctly	(A0) rounded to 2 sf)
		(vi)	7.43 (inc	correctly	(A0) rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
(a) BC = $\sqrt{32^2 - 30^2}$ (M1) Award (M1) for correct substitution in Pythagoras' formula = 11.1 ($\sqrt{124}$,11.1355)(cm) (A1)	(a) BC = $\sqrt{32^2 - 30^2}$ 11 (cm) (2 sf answer on	(M1) (A1) ly seen, but correct)
(b) Area = $\frac{1}{2} \times 30 \times 11.1355$ (M1) Award (M1) for correct substitution in area of triangle formula	(b) case (i) Area = $\frac{1}{2} \times 30 \times 11$	(M1) (working shown)
$=167(167.032)(cm^2)$ (A1)(ft)	$=165 (\mathrm{cm}^2)$	<i>(A1)</i> (ft)
	case (ii) =165 (cm ²) (No working show treated as a ft, so no m	(M0)(A0)(ft) wn, the answer 11 is harks awarded here)

Example: ABC is a right angled triangle with angle $ABC = 90^{\circ}$, AC = 32 cm and AB = 30 cm. Find (a) the length of BC, (b) The area of triangle ABC.

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (*eg*, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (*MO*).

Marking

(A2)

(A1)

(A1)

(A0)

(A0)

(A0)

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

Markschem	е		Candidates' Scripts
Chi-squared		(a)	7.7
7.68 (7.67543)	(A2)	(b)	7.67

7.6

8

7

7.66

(c)

(d)

(e)

(e)

eg, Chi-squared, correlation coefficient, mean

Regression line

Markscheme		Candidates' Scripts	Marking
y = 0.888x + 13.5 (A2) ($y = 0.887686x + 13.4895$)	(a)	y = 0.89x + 13	(A2) (both accepted)
If an answer is not in the form of an equation award at most (A1)(A0).	(b)	y = 0.88x + 13	(A1) (one rounding error)
	(c)	y = 0.88x + 14	(A1) (rounding error repeated)
	(d)	(i) $y = 0.9x + 13$	
		(ii) $y = 0.8x + 13$	(A1) (1 sf not accepted)
	(e)	0.88x + 14 (two round	(A0) ling errors and not an equation)

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
(2.06, 4.49) (A1)(A1) (2.06020, 4.49253)	(a) (2.1, 4.5)	(A1)(A1) (both accepted)
	(b) (2.0, 4.4) (sar	(A1) me rounding error twice)
	(c) (2.06, 4.4)	(A1) (one rounding error)
	(d) (2, 4.4) (1sf not accep	(A0) ted, one rounding error)

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final A mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme			Marking		
\$231.62 (231.6189)	(A1)	(i)	231.6		(A0)
		(ii)	232	(Correct round	(A0) ing to incorrect level)
		(iii)	231.61		(A0)
		(iv)	232.00		(A0) (iii) and (iv) are both ding to correct level)

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.

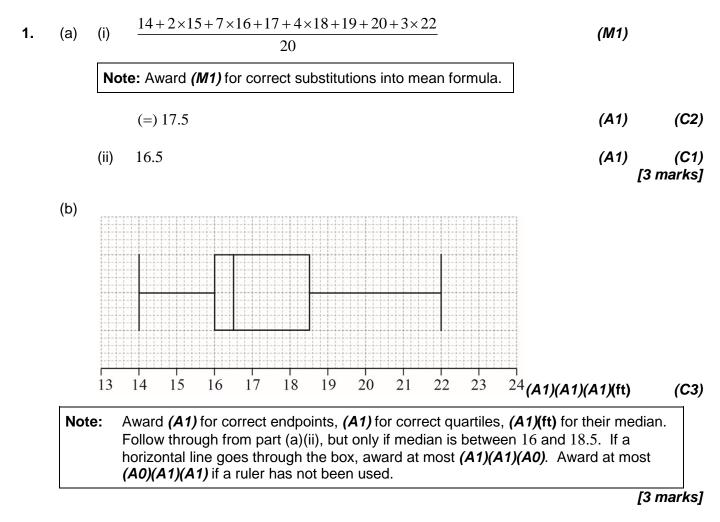
The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

	Markschem	е		Candidates' Scripts	Marking	
(a)	$37000\mathrm{m}^2$	(A1)	(a)	36000 m ² (Incorre	ect answer so u	(A0) Inits not considered)
(b)	3200 m ³	(A1)	(b)	3200 m^2		(A0) (Incorrect units)

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.



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Total [6 marks]
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(a)
$$(0, -2.5)$$
 OR $\left(0, -\frac{5}{2}\right)$ (A1)(A1) (C2)
Note: Award (A1) for 0 and (A1) for -2.5 written as a coordinate pair. Award at most (A1)(A0) if brackets are missing. Accept " $x = 0$ and $y = -2.5$ ".
[2 marks]
(b) $\frac{2-(-7)}{-6-6}$ (M1)
Note: Award (M1) for correct substitution into gradient formula.
 $= -\frac{3}{4}$ (-0.75) (A1) (C2)
[2 marks]
(c) (i) $\frac{4}{3}$ (1.33333...) (A1)(ft) (C1)
Note: Award (A0) for $\frac{1}{0.75}$. Follow through from part (b).
(ii) $y = \frac{4}{3}x - \frac{5}{2}$ ($y = 1.33...x - 2.5$) (A1)(ft) (C1)
Note: Follow through from parts (c)(i) and (a). Award (A0) if final answer is not written in the form $y = mx + c$.
[2 marks]

2.

(a)	$\frac{149600000}{300000 \times 60}$	(M1)(M1)
Not	te: Award (<i>M1</i>) for dividing the correct numerator (which c form such as 149.6×10^6 or 1.496×10^8) by 300000 and	-
	= 8.31 (minutes) (8.31111, 8 minutes 19 seconds)	(A1) (C3) [3 marks]
(b)	323×9467280	(M1)
Not	te: Award (M1) for multiplying 323 by 9467280, seen with only penalizing incorrect power of 10 once.	any power of 10; therefore
	$= 3.06 \times 10^9 \ (= 3.05793 \times 10^9)$	(A1)(A1) (C3)
Not	te:Award (A1) for 3.06. Award (A1) for $\times 10^9$. Award (A0)(A0) for answers of the type: 30.6×10^8 .	
L		[3 marks]

4. (a) if the car is less than 2 years old or the car has not been driven more than 20000 km, then the car is under warranty (A1)(A1)(A1) (C3)

Note: Award *(A1)* for if ..., then ..., *(A1)* for "or", *(A1)* for correct statements in correct order. Accept "If the car has not been driven more than 20000 km or the car is less than 2 years old, then the car is under warranty". Accept logical equivalent wording for each proposition, *eg* "less than 20000 km".

[3 marks]

(b)

р	q	r	$\neg r$	$q \lor \neg r$	$(q \vee \neg r) \Longrightarrow p$
Т	Н	Т	F	Т	Т
Т	Т	F	Т	т	Т
Т	F	Т	F	F	Т
Т	F	F	Т	Т	Т
F	Т	Т	F	Т	F
F	Т	F	Т	Т	F
F	F	Т	F	F	т
F	F	F	Т	Т	F

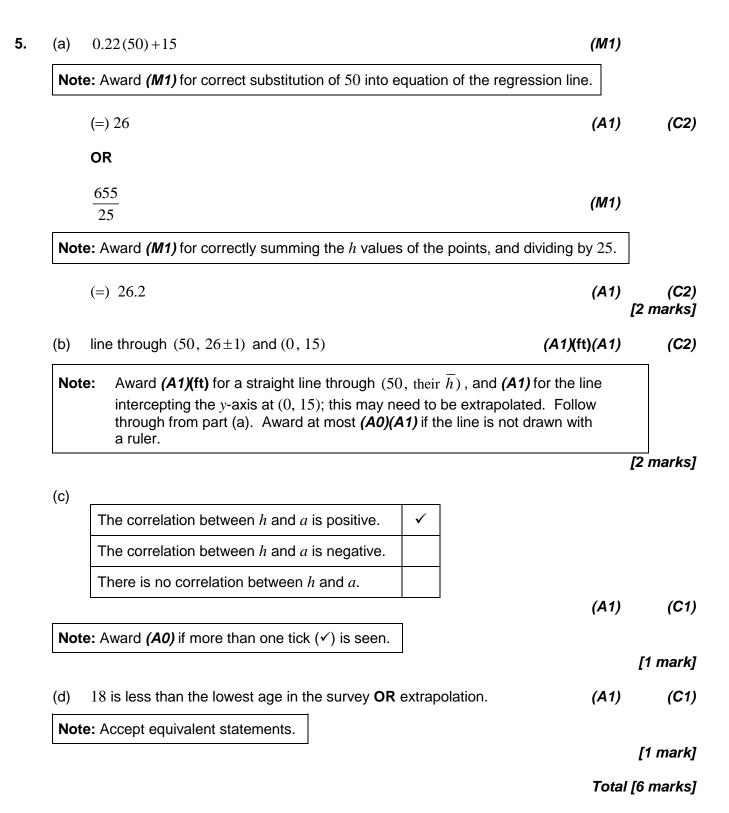
(A1)(A1)(ft) (C2)

Note: Award **(A1)** for $q \lor \neg r$ column correct and **(A1)(ft)** for $(q \lor \neg r) \Rightarrow p$ column correct. Follow through from their $q \lor \neg r$ column.

[2 marks]

(c) contrapositive

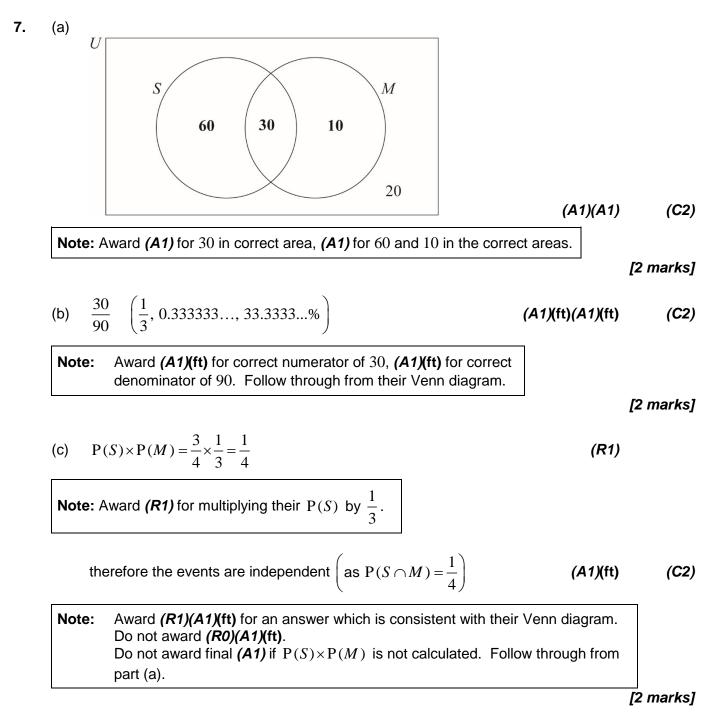
(A1) (C1) [1 mark]



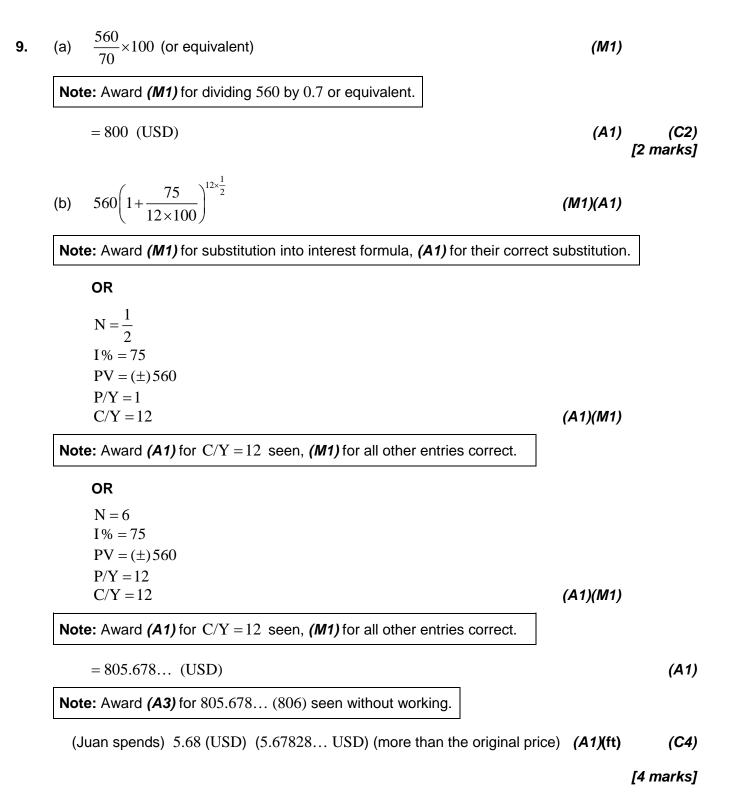
(a) $x^2 + y^2 = 68^2$ (or 4624 or equivalent) 6. (A1) (C1) [1 mark] (b) $\frac{y}{x} = \frac{3}{4}$ (M1) Note: Award (M1) for a correct equation. $y = \frac{3}{4}x$ (y = 0.75x) (A1) (C2) [2 marks] (c) $x^{2} + \left(\frac{3}{4}x\right)^{2} = 68^{2}$ (or $x^{2} + \frac{9}{16}x^{2} = 4624$ or equivalent) (M1) Note: Award (M1) for correct substitution of their expression for y into their answer to part (a). Accept correct substitution of x in terms of y. x = 54.4 (cm), y = 40.8 (cm) (A1)(ft)(A1)(ft) (C3) Follow through from parts (a) and (b) as long as x > 0 and y > 0. Note: [3 marks]

Total [6 marks]

– 16 –



(a) $\frac{350}{0.1559}$	(M1)	
Note: Award <i>(M1)</i> for dividing 350 by 0.1559.		
= 2245.03 (ARS)	(A1)	[2 m
(b) 2245.03×1.02	(M1)	
Note: Award <i>(M1)</i> for multiplying their answer to part (a) by 1.02.		
= 2289.93 (ARS)	<i>(A1)</i> (ft)	
OR		
2245.03×0.02	(M1)	
Note: Award <i>(M1)</i> for multiplying their answer to part (a) by 0.02.		
= 44.9006 2245.03 + 44.90		
= 2289.93 (ARS)	<i>(A1)</i> (ft)	
Note: Follow through from part (a).		[2 m
(c) $\frac{4228.38}{585}$	(M1)	



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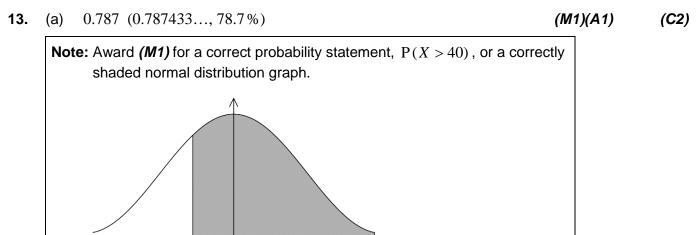
$\frac{\text{EB}}{\sin 53^{\circ}} = \frac{1.2}{\sin 7^{\circ}}$ (a) (M1)(A1) Note: Award (M1) for substitution into sine formula, (A1) for correct substitution. (EB =) 7.86 m **OR** 786 cm (7.86385...m **OR** 786.385...cm) (A1) (C3) [3 marks] (b) 34° (A1) (C1) [1 mark] $\sin 34^\circ = \frac{\text{height}}{7.86385...}$ (c) (M1) Note: Award (M1) for correct substitution into a trigonometric ratio. (height =) 4.40 m OR 440 cm (4.39741... m OR 439.741... cm) (A1)(ft) (C2) Accept "BT" used for height. Follow through from parts (a) and (b). Use of 7.86 Note: gives an answer of 4.39525.... [2 marks]

10. Units are required in parts (a) and (c).

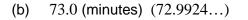
11.	(a)	x = -2	(A1)(A1)	(C2)
	Not	e: Award (A1) for $x =$ (a constant) and (A1) for -2 .		
				[2 marks]
	(b)	(<i>c</i> =) 5	(A1)	(C1) [1 mark]
	(c)	$-\frac{b}{2a} = -2$		
		$a(-2)^2 - 2b + 5 = 3$ or equivalent		
		$a(-4)^2 - 4b + 5 = 5$ or equivalent		
		2a(-2)+b=0 or equivalent	(M1)	
	Not	e: Award (M1) for two of the above equations.		
		<i>a</i> = 0.5	<i>(A1)</i> (ft)	
		<i>b</i> = 2	<i>(A1)</i> (ft)	(C3)
	Not	e: Award at most (M1)(A1)(ft)(A0) if the answers are re Follow through from parts (a) and (b).	versed.	
	L			[3 marks]

[3 marks]

(a)	the temperature in the oven	(A1)			
	OR				
	the maximum possible temperature of the cake mix	(A1)	(C1)		
N	ote: Award (A0) for "the maximum temperature".				
			[1 mark]		
(b)	$18 = 150 - a(\times 1.1^{\circ})$	(M1)			
N	Note: Award <i>(M1)</i> for correct substitution of 18 and 0. Substitution of 0 can be implied.				
	(a) = 132	(A1) [2	(C2) ? marks]		
(c)	$150 - 132 \times 1.1^{-t} = 130$	(M1)			
N	Dete: Award (M1) for substituting their <i>a</i> and equating to 130. Accept an Award (M1) for a sketch of the horizontal line on the graph.	n inequality.			
	t = 19.8 (19.7992)	<i>(A1)</i> (ft)			
N	ote: Follow through from part (b).				
	34.8 (minutes) (34.7992, 34 minutes 48 seconds)	<i>(A1)</i> (ft)	(C3)		
N	bte: Award the final (A1) for adding 15 minutes to their <i>t</i> value. In part (c), award (C2) for a final answer of 19.8 with no working.				
		່ <i>[</i> ິ	8 marks]		



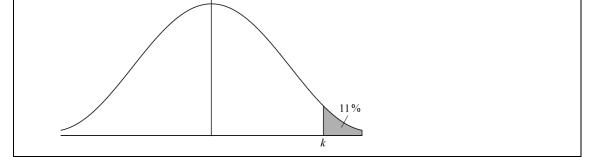
[2 marks]



40

(M1)(A1) (C2)

Note: Award *(M1)* for a correct probability statement, P(X > k) = 0.11, or a correctly shaded normal distribution graph.



[2 marks]

(c) 0.0423433...×400

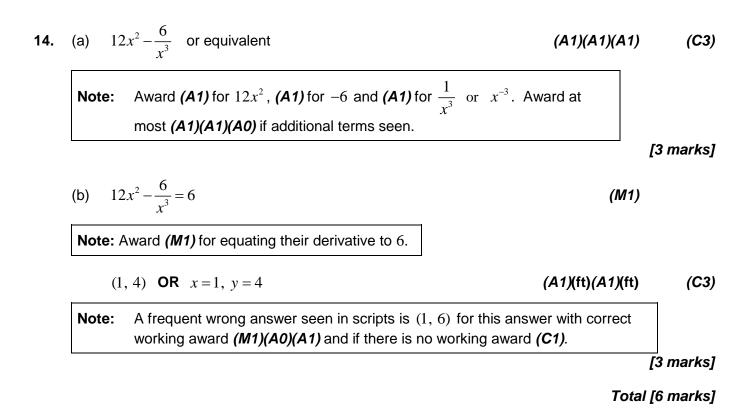
Note: Award *(M1)* for multiplying a probability by 400. Do not award *(M1)* for 0.11×400 . Use of a lower bound less than zero gives a probability of 0.0429172...

(A1) (C2)

(M1)

Notes: Accept a final answer of 17. Do not accept a final answer of 18. Accept a noninteger final answer either 16.9 (16.9373...) from use of lower bound zero or 17.2 (17.1669...) from use of the default lower bound of $-10^{.99}$.

[2 marks]



5.	(a)	522 (kg)	(A1)	(C1) [1 mark]
	(b)	522(8-6.80) or equivalent	(M1)	
	Not	te: Award (M1) for multiplying their answer to part (a) by $(8-6.80)$.		
		626 (EUR) (626.40)	<i>(A1)</i> (ft)	(C2)
	Not	e: Follow through from part (a).		
	(c)	(W =) (882 - 45p)(p - 6.80)	(A1)	[2 marks]
		OR		
		$(W =) - 45p^2 + 1188p - 5997.6$	(A1)	(C1) [1 mark]
	(d)	sketch of W with some indication of the maximum	(M1)	
		OR		
		-90p+1188=0	(M1)	
	Not	e: Award (M1) for equating the correct derivative of their part (c) to zero.]	
		OR		
		$(p =) \frac{-1188}{2 \times (-45)}$	(M1)	
	Not	e: Award (M1) for correct substitution into the formula for axis of symmetry	y.	
		(p =) 13.2 (EUR)	<i>(A1)</i> (ft)	(C2)
	Not	te: Follow through from their part (c), if the value of p is such that $6.80 < p$	<19.6.	
	L			[2 marks]
			Tota	l [6 marks]