Types of numbers

You have to be able to classify numbers as natural numbers, integers, rational numbers and real numbers.

The presentation will cover several examples of problems involving the classification of numbers. Before you go through those examples, make sure that you study chapter 1.1 from your textbook (pages 2-7)

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An example of an exam question:

Consider the numbers $\sqrt{3}$, 6, $2\frac{1}{2}$, π, -5, and the sets $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$. Complete the following table by placing a tick in the appropriate box if the number is an element of the set.

	$\sqrt{3}$	6	$2\frac{1}{2}$	π	-5
$\mathbb{N}$					
$\mathbb{Z}$					
$\mathbb{Q}$					

(Total 6 marks)

The first observation here is that: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ ie. every natural number is an integer, every integer is a rational number and every rational number is a real number.

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So for instance since 6 is obviously a natural number, then you should place a tick in every box under the 6 (because since it's natural, then it's also integer and rational).

Tricky example

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> -5 is a rational number, because it can be written as $\frac{-5}{1}$,
> $2\frac{1}{2}$ is a rational number, because it can be written as $\frac{5}{2}$.

Note that it has to be a fraction of two **integers**. For instance $\frac{\sqrt{2}}{2}$ is a fraction, but it's not a fraction of two integers and can never be written as such. Hence $\frac{\sqrt{2}}{2}$ is not a rational number.

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Main definition:

Definition 1

A number is rational if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

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Main definition:

Definition 1

A number is rational if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Alternative definition:

Definition 2

A number is rational if it has a finite or recurring decimal expansion.

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In fact:

$0.77777... = \frac{7}{9}$, and
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This second definition is important, when we see numbers like 0.7777... or 0.12121212.... These are both rational numbers.

In fact:

$0.77777... = \frac{7}{9}$, and

$0.12121212... = \frac{4}{33}$.

But you don't have to convert them into fractions. It is enough that you notice that they have recurring decimal expansion (czyli okresowe rozwinięcie dziesiętne).

Irrationals

Third important thing is to be familiar with typical irrational numbers. These include: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and π.

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Third important thing is to be familiar with typical irrational numbers. These include: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and π.

Be careful however. Just because a number has a $\sqrt{}$ sign, does not mean that it is immediately classified as irrational. For instance $\sqrt{4}$ is of course a natural number (since $\sqrt{4} = 2$). A less obvious example would be $\frac{\sqrt{8}}{\sqrt{2}}$, but again this is a natural number (it is also equal to 2).

The next slides will include first a solution to the exam question and then two more question (question on one slide, solution on the next).

Solution

Consider the numbers $\sqrt{3}$, 6, $2\frac{1}{2}$, π, -5, and the sets $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$. Complete the following table by placing a tick in the appropriate box if the number is an element of the set.

	$\sqrt{3}$	6	$2\frac{1}{2}$	π	-5
$\mathbb{N}$		✓			
$\mathbb{Z}$		✓			✓
$\mathbb{Q}$		✓	✓		✓

(Total 6 marks)

Question 1

Consider the numbers 2, $\sqrt{3}$, $-\frac{2}{3}$ and the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$.

Complete the table below by placing a tick in the appropriate box if the number is an element of the set, and a cross if it is not.

		$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
(i)	2				
(ii)	$\sqrt{3}$				
(iii)	$-\frac{2}{3}$				

(3)

Solution

Consider the numbers 2, $\sqrt{3}$, $-\frac{2}{3}$ and the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$.

Complete the table below by placing a tick in the appropriate box if the number is an element of the set, and a cross if it is not.

		$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
(i)	2	✓	✓	✓	✓
(ii)	$\sqrt{3}$	✗	✗	✗	✓
(iii)	$-\frac{2}{3}$	✗	✗	✓	✓

(3)

Question 2

Consider the numbers 5, 0.5, $\sqrt{5}$ and -5. Complete the table below, showing which of the number sets, $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{Q}$ these numbers belong to.

Answers:

	$\mathbb{N}$	$\mathbb{R}$	$\mathbb{Q}$
5			✓
0.5	✗		
$\sqrt{5}$	✗		
-5		✓	

(Total 8 marks)

Solution

Consider the numbers 5, 0.5, $\sqrt{5}$ and -5. Complete the table below, showing which of the number sets, $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{Q}$ these numbers belong to.

Answers:

	$\mathbb{N}$	$\mathbb{R}$	$\mathbb{Q}$
5	✓	✓	✓
0.5	✗	✓	✓
$\sqrt{5}$	✗	✓	✗
-5	✗	✓	✓

(Total 8 marks)

The short test and the beginning of the class will be similar to the questions above.

In case of any questions you can email me at T.J.Lechowski@gmail.com.