N14/5/MATSD/SP1/ENG/TZ0/XX/M



International Baccalaureate<sup>®</sup> Baccalauréat International Bachillerato Internacional

# MARKSCHEME

### November 2014

## **MATHEMATICAL STUDIES**

### **Standard Level**

Paper 1

24 pages

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#### Paper 1 Markscheme Instructions to Examiners

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Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6.

#### Abbreviations 1

The markscheme may make use of the following abbreviations:

- M Marks awarded for **Method**
- A Marks awarded for an Answer or for Accuracy
- С Marks awarded for **Correct** answers (irrespective of working shown)
- R Marks awarded for clear Reasoning
- Marks that can be awarded as **follow through** from previous results in the question ft

#### 2 **Method of Marking**

- All marking must be done in RM Assessor using the mathematical studies annotations and in (a) accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- If the candidate has full marks on a question use the C6 annotation, if the candidate has made (b) an attempt but scores zero marks use C0. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.
- (c) In this paper, if the correct answer is seen on the answer line the maximum mark is awarded. There is no need to check the working! Award C marks and move on.
- (d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.
- If the **answer is wrong**, marks should be awarded for the working according to the markscheme. (e)
- Working crossed out by the candidate should not be awarded any marks. Where candidates have (f) written two solutions to a question, only the first solution should be marked.
- A correct answer in the working box transcribed inaccurately to the answer line can receive full marks. (g)
- If correct working results in a correct answer in the working box but then further working (h) is developed, full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer as long as it is unambiguous.

Accuracy of numerical answers is an exception to this rule – see Section 5.

**Example:** Factorise  $x^2 - 5x - 6$ 

Markscheme		Candidates' Scripts		Marking	
(x-6)(x+1)	(A1)(A1)	(i)	Answer line: $(x+6)(x+1)$	(A0)(A1)	
		(ii)	Working box: $(x-6)(x+1)$ followed by $x=6$ and $-1$ , or just $6, -1$	(A1)	
			in either working box or on answer line.	(A0)	

#### **3** Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(**ft**)'.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final *A* mark should not be awarded.
- (c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

	Markscheme	Candidates' Scripts		Marking
(a)	$\frac{\sin A}{3} = \frac{\sin 30}{4} \qquad (M1)(A1)$	(a)	$\frac{\sin A}{4} = \frac{\sin 30}{3}$	( <b>M1</b> )( <b>A0</b> ) (use of sine rule but with wrong values)
	$A = 22.0^{\circ} (22.0243)$ (A1)		and cannot be a	(A0) 1) here was not marked (ft) warded because there was n the same question part.)
(b)	$x = 7 \tan (22.0243^{\circ})  (M1)$ = 2.83 (2.83163) (A1)(ft)	(b) <i>but</i>	case (i) $x = 7 \tan 41.8^{\circ}$ = 6.26 case (ii) 6.26	(M1) (A1)(ft) (C0) since no working shown

#### 4 Using the Markscheme

(a) A marks are **dependent** on the preceding M mark being awarded, it is **not** possible to award (M0)(A1). Once an (M0) has been awarded, all subsequent A marks are lost in that part of the question, even if calculations are performed correctly, until the next M mark. The only exception will be for an answer where the accuracy is specified in the question – see section 5.

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- (b) A marks are **dependent** on the R mark being awarded, it is **not** possible to award (A1)(R0). Hence the (A1) is not awarded for a correct answer if no reason or the wrong reason is given.
- (c) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

(d) Unless the question specifies otherwise, accept equivalent forms. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value (for example  $\frac{2}{3}$  if applicable);

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(e) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1.7; 1,7.

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49.

Different descriptions of an interval: 3 < x < 5; (3, 5); ] 3, 5 [.

Different forms of notation for set properties (*e.g.* complement): A';  $\overline{A}$ ;  $A^c$ ; U - A;  $(A; U \setminus A)$ . Different forms of logic notation:  $\neg p$ ; p';  $\overline{p}$ ;  $\sim p$ .

$$p \Rightarrow q; p \rightarrow q; q \leftarrow p.$$

Significance level may be written as  $\alpha$ .

(f) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

### 5 Accuracy of Answers

#### Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award (*A1*) and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

- 2. If the candidate's unrounded answer is **not** seen then award (A1) if the answer given is **correctly** rounded to 2 or more significant figures, otherwise (A0).
- **Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
- **3.** If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

	If candidates final answer is given							
	Exact or to 4 or more sf (and would <b>round</b> <b>to the correct 3 sf</b> )	Correct to 3 sf	Incorrect to 3 sf	Correct to $2 \text{ sf}^3$	Incorrect to 2 sf	Correct or incorrect to 1 sf		
Unrounded answer seen <sup>1</sup>	Award the	Award the final (A1) irrespective of correct or incorrect rounding						
Unrounded answer not seen <sup>2</sup>	(A1)	(A1)	(A0)	(A1)	(A <b>0</b> )	(A0)		
Treatment of subsequent parts	As per MS	Treat as fo	ollow through	, only if wor	king is seen. <sup>3</sup>			

Examples:

Markscheme			Candidates' Scripts		Marking
9.43 (9.43398)	(A1)	(i)	9.43398 is seen in the working followed by 9; 9.4; 9.43; 9.434 (correctly rounded)		(A1)
		(ii)	9.43398 is seen in the working followed by 9.433; 9.44 <i>etc</i> . (incorre rounded)		(A1)
		(iii)	9.4		(A1)
		(iv)	9		(A0) (correct to 1 sf)
		(v)	9.3 (ii	ncorrectly	(A0) rounded to 2 sf)
		(vi)	9.44 (ii	ncorrectly	(A0) rounded to 3 sf)

Markscheme			Candidates' Scripts	Marking
7.44 (7.43798)	(A1)	(i)	7.43798 is seen in the working bo followed by 7; 7.4; 7.44; 7.438 <i>etc</i> (correctly rounded)	
		(ii)	7.43798 is seen in the working bo followed by 7.437; 7.43 <i>etc</i> . (incorrectl rounded)	
		(iii)	7.4	(A1)
		(iv)	7	( <b>A0</b> ) (correct to 1 sf)
		(v)	7.5 (ince	( <b>A0</b> ) orrectly rounded to 2 sf)
		(vi)	7.43 (inc	( <b>A0</b> ) orrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
(a) BC = $\sqrt{32^2 - 30^2}$ (M1) Award (M1) for correct substitution in Pythagoras' formula =11.1 ( $\sqrt{124}$ ,11.1355)(cm) (A1)	(a) BC = $\sqrt{32^2 - 30^2}$ 11 (cm) (2 sf answer on	(M1) (A1) ly seen, but correct)
(b) Area = $\frac{1}{2} \times 30 \times 11.1355$ (M1) Award (M1) for correct substitution in area of triangle formula	(b) case (i) Area = $\frac{1}{2} \times 30 \times 11$	( <b>M1</b> ) (working shown)
$=167(167.032)(cm^2)$ (A1)(ft)	$=165 (cm^2)$	(A1)(ft)
	case (ii) = $165 (\text{cm}^2)$ (No working show treated as a ft, so no m	( <b>M0)(A0)(ft)</b> wn, the answer 11 is warks awarded here)

**Example:** ABC is a right angled triangle with angle  $ABC = 90^{\circ}$ , AC = 32 cm and AB = 30 cm. Find (a) the length of BC, (b) The area of triangle ABC.

Rounding of an exact answer to 3 significant figures should be accepted if performed correctly. Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (*e.g.*  $\sqrt{13}$ ,  $2^{\frac{2}{3}}$ ,  $\sqrt[4]{5}$ ,) may be accepted as exact answers. All other powers (*eg*, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (M0).

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Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	(A2)
7.68 (7.67543) (A2)	(b) 7.67	(A1)
	(c) 7.6	(A1)
	(d) 8	(A <b>0</b> )
	(e) 7	(A <b>0</b> )
	(e) 7.66	(A0)

eg, Chi-squared, correlation coefficient, mean

Regression line

Markscheme		Candidates' Scripts	Marking
y = 0.888x + 13.5 (A2) ( $y = 0.887686x + 13.4895$ )	(a)	y = 0.89x + 13	( <b>A2</b> ) (both accepted)
If an answer is not in the form of an equation award at most $(A1)(A0)$ .	(b)	y = 0.88x + 13	(A1) (one rounding error)
	(c)	y = 0.88x + 14	(A1) (rounding error repeated)
	(d)	(i) $y = 0.9x + 13$	
		(ii) $y = 0.8x + 13$	(A1) (1 sf not accepted)
	(e)	0.88 <i>x</i> + 14 <i>(two roundi</i>	( <b>A0</b> ) ing errors and not an equation)

Markscheme		Candidates' Sc	ripts Marking
(2.06, 4.49) (A1)(A1) (2.06020, 4.49253)	(a)	(2.1, 4.5)	(A1)(A1) (both accepted)
	(b)	(2.0, 4.4)	(A1) (same rounding error twice)
	(c)	(2.06, 4.4)	(A1) (one rounding error)
	(d)	(2, 4.4)	(A0) (1sf not accepted, one rounding error)

Maximum/minimum/points of intersection

#### 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final A mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example:	A financial question demands accuracy correct to 2 dp.
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Markscheme			Candidates' Scripts	Marking
\$231.62 (231.6189) (A1	) (i)	231.6		(A0)
	(ii)	232	(Correct rounding	( <b>A0</b> ) ag to incorrect level)
	(iii)	231.61		(A0)
	(iv)	232.00		( <b>A0</b> ) iii) and (iv) are both ling to correct level)

#### 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final A mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

Markscheme			Candidates' Scripts			Marking
(a)	$37000 \text{ m}^2$	(A1)	(a)	36000 m <sup>2</sup> ( <i>Incon</i>	rrect answer so u	( <b>A0</b> ) nits not considered)
(b)	3200 m <sup>3</sup>	(A1)	(b)	3200 m <sup>2</sup>		( <b>A0</b> ) (Incorrect units)

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

#### 8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

1.	(a)	(i) 300 (minutes) <b>OR</b> 5 hours	(A1)	
		<b>Note:</b> If answer given in hours, the unit <b>must</b> be seen.		
		(ii) 220–100	(M1)	
		Notes: Award (M1) for the two quartiles seen.		
		= $120$ (minutes) <b>OR</b> 2 hours	(A1)	(C3)
		<b>Note:</b> If answer given in hours, the unit <b>must</b> be seen.		
	(b)	median (time spent on homework per day)	(A1)	(C1)
		<b>Note:</b> Do not accept middle or medium etc.		
	(c)	(i) 25	(A1)	
		(ii) 75	(A1)	(C2)
			[6	marks]

2. (a) 
$$BD = \sqrt{(4^2 + 8^2)}$$
 (M1)  
Note: Award (M1) for correct substitution into the distance formula.  

$$= 8.94 \left(8.94427..., \sqrt{80}, 4\sqrt{5}\right)$$
(A1) (C2)  
(b) Area ABCD =  $2 \times \left(0.5 \times \frac{\text{their BD}}{2} \times \sqrt{500}\right)$  (M1)(M1)(M1)  
Note: Award (M1) for dividing their BD by 2, (M1) for correct substitution into the area of triangle formula, (M1) for adding two triangles (or multiplied by 2).  
Accept alternative methods:  
Area of kite =  $0.5 \times \sqrt{500} \times \text{their part (a)}$ .  
Award (M1) for correctly substituting in  $\sqrt{500}$ .  
Award (M1) for correctly substituting in their part (a).

=100

(A1) (C4)

Note: Accept 99.9522 if 3 sf answer is used from part (a).

[6 marks]

3.	(a)	discrete	(A1)	(C1)
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- (b) (i) 60 (A1)
  - (ii) 5 (A1) (C2)

(c) (i) 
$$\frac{1 \times 4 + 2 \times 7 + 3 \times 12...}{60}$$
 (M1)

Notes: Award (M1) for an attempt to substitute into the "mean of a set of data" formula, with at least three correct terms in the numerator. Denominator must be 60.Follow through from part (b)(i), only if work is seen.

= 4.03 (4.03333...)

(A1)

Notes: Award at most (M1)(A0) for an answer of 4 but only if working seen.

(ii) 1.54 (1.53803...)

(A1) (C3) [6 marks]

**4.** (a) 
$$\frac{0.97 \times 3550}{1.95}$$

#### (M1)(M1)(M1)

Note: Award (*M1*) for 0.97 seen, (*M1*) for 0.97×3550, (*M1*) for division by 1.95.

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OR

$(3550 - 0.03 \times 3550) \times -$	1
	1.95

(M1)(M1)(M1)

(A1)

(M1)

(A1)

**Note:** Award (*M1*) for 0.03×3550 seen, (*M1*) for subtracting 0.03×3550 from 3550, (*M1*) for division by 1.95.

=1765.90 (EUR)

(b)  $20 \times 1.90 - 34.50$ 

Note: Award (M1) for subtraction of 34.50 from their product of  $20 \times 1.90$ .

= 3.50 (BGN)

Notes: Award at most (M1)(A0) for an answer of 4, but only if working seen.

[6 marks]

*(C4)* 

(C2)

Note: Award

Not	For	: Spani		hen" , <b>only</b> accept "Si" and "en reak my arm" and "not h			
(b)	р	q	$p \Rightarrow q$	Inverse of $p \Rightarrow q$	Converse of $p \Rightarrow q$		
	Т	Т	Т	Т	Т		
	Т	F	F	Т	Т		
	F	Т	Т	F	F		
	F	F	Т	Т	Т		
<ul> <li>Notes: Award (A1) for each correct column.</li> <li>(c) logically equivalent last two columns of the truth table are identical</li> </ul>					(A1)(A1) (A1)(ft) (R1)(ft)	(C.	
Note			ard ( <b>A1</b> )( <b>ft</b> )( <b>I</b> rough from t	<b>R0</b> ). he last two columns of th	ne table in part (a).		

6. (a) 
$$FV = 900000 \left( 1 + \frac{15}{12 \times 100} \right)$$

(M1)(A1)

Note: Award (*M1*) for substitution in the compound interest formula (either FV or interest), do not penalize if -PV not seen. Award (*A1*) for correct substitution.

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### OR

N = 2 I% = 15 PV = 900000 P/Y = 1C/Y = 12

(A1)(M1)

Note: Award (A1) for C/Y = 12 seen, (M1) for other correct entries.

### OR

N = 24	
I% =15	
PV = 900000	
P/Y = 12	
C/Y = 12	(A1)(M1)

Note: Award (A1) for C/Y = 12 seen, (M1) for other correct entries.

interest = 312615.945	(A1)	
= 312600 (KHR)	(A1)(ft)	(C4)

Notes: Award the final (A1) for the correct rounding of their unrounded answer. If final amount is 1212600 and working is shown award (M1)(A1)(A0)(A1)(ft). Award (A2) for (FV =) 1212600 if no working is seen.

(b)  $3.126 \times 10^5$ 

Notes: Award (A1)(ft) for their 3.126 (3.13), (A1)(ft) for  $\times 10^5$ . Follow through from part (a).

[6 marks]

(C2)

(A1)(ft)(A1)(ft)

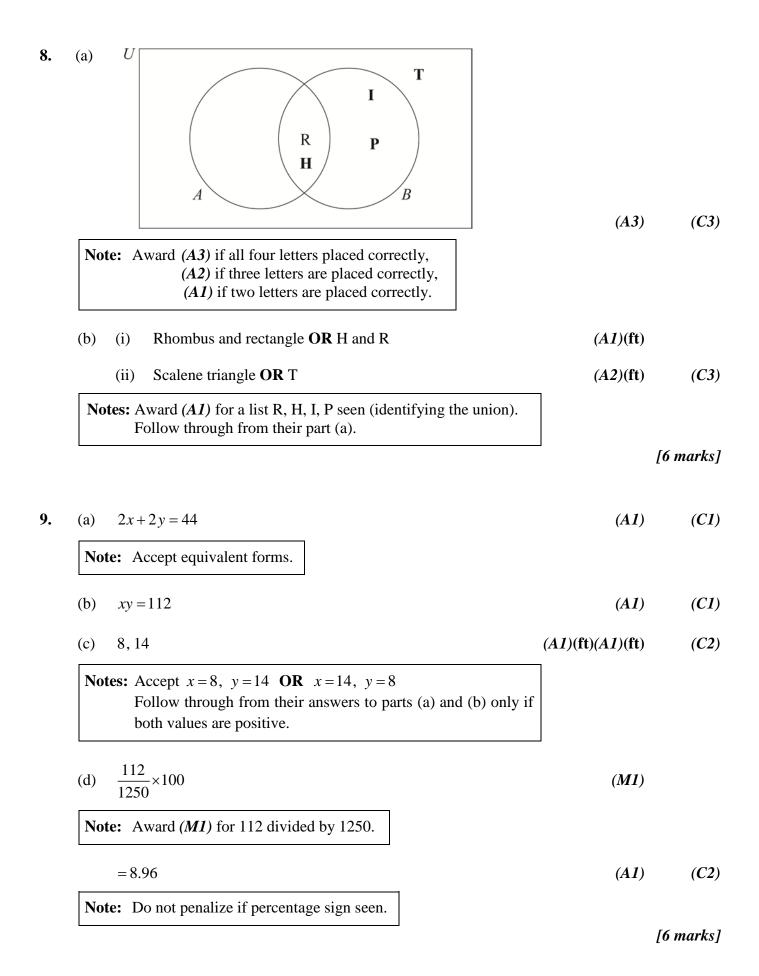
7. (a) 
$$\frac{600-150}{6-1}$$
 (M1)

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OR

600 = 150 + (6 - 1)m	(M1)	
<b>Note:</b> Award <i>(M1)</i> for correct substitution into gradient formula or arithmetic sequence formula.		
= 90	(A1)	(C2)
<ul><li>(b) the annual rate of growth of the number of apartments</li><li>Note: Do not accept common difference.</li></ul>	(A1)	(C1)
(c) $150 = 90 \times (1) + n$	(M1)	
<b>Note:</b> Award ( <i>M1</i> ) for correct substitution of their gradient and one of the given points into the equation of a straight line.		
n = 60	(A1)(ft)	(C2)
<b>Note:</b> Follow through from part (a).		
(d) the initial number of apartments	(A1)	(C1)
<b>Note:</b> Do not accept "first number in the sequence".		

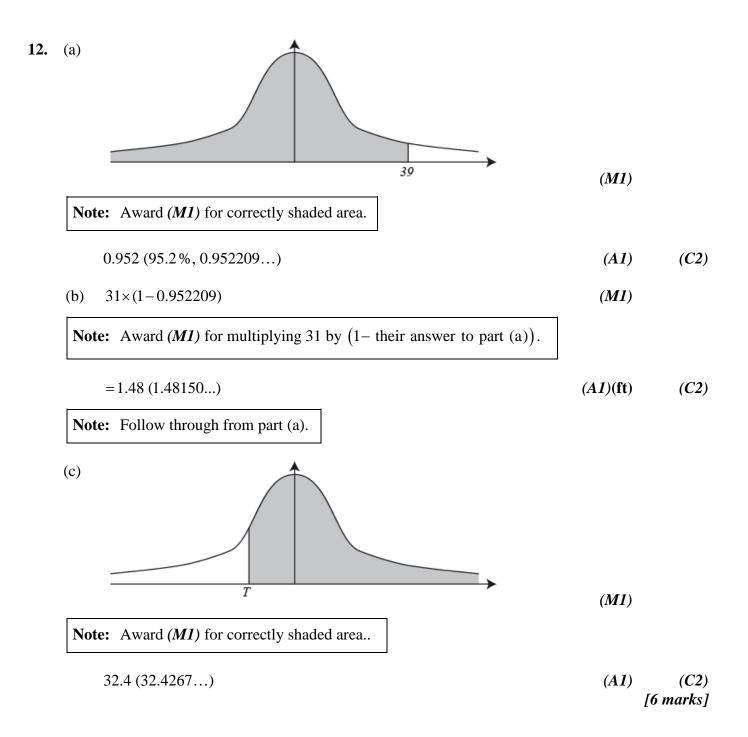
[6 marks]



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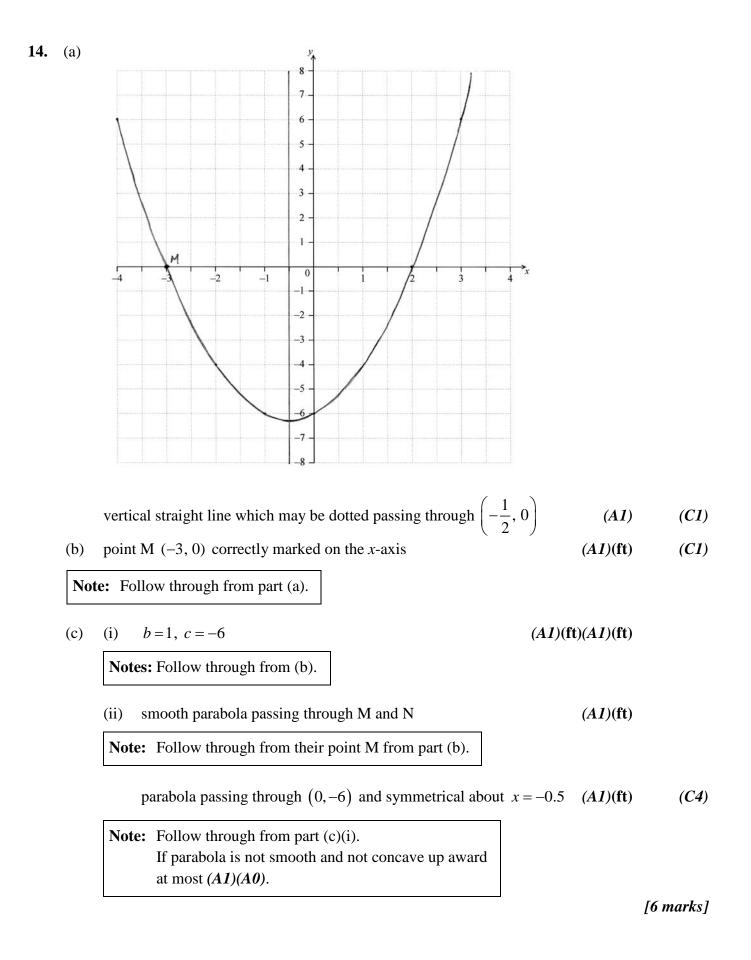
10.	(a)	$H_0$ : Choice of morning snack is independent of (not dependent of gender.	n) (A1)	(C1)
	No	<b>te:</b> Accept there is "no association" between snack chosen and gender. Do not accept "not related" or "not correlated" or "influenced".		
	(b)	2	(A1)	(C1)
	(c)	$\frac{210\times150}{350}$	(M1)	
	No	<b>te:</b> Award ( <i>M1</i> ) for correct substitution in the correct formula.		
		= 90	(A1)	(C2)
	(d)	Null hypothesis is accepted (not rejected).	(A1)	
		OR		
		Choice of morning snack is independent of gender	(A1)	
		3.576 < 5.99 <b>OR</b> $\chi^2_{calc} < \chi^2_{crit}$	( <b>R</b> 1)	(C2)
	No	te: Do not award $(A1)(R0)$ .		
				[6 marks]
11.	(a)	(i) (-1, 0)	(A1)	
	No	te: Accept –1.		
		(ii) (0,-1)	(A1)	(C2)
	No	te: Accept –1.		
	(b)	(x =) -2.96 (-2.96135) (x =) 1.34 (1.33508)	(A1) (A1)	(C2)
	(c)	-2.96 < <i>x</i> <1.34 <b>OR</b> ]-2.96, 1.34[ <b>OR</b> (-2.96, 1.34)	(A1)(ft)(A1)	(C2)
	No	<b>tes:</b> Award ( <i>A1</i> )( <b>ft</b> ) for both correct endpoints of the interval, ( <i>A1</i> ) correct strict inequalities (or correct open interval notation). Follow through from part (b).	for	

[6 marks]



## 13. For parts (a) and (b) only, the first time a correct answer has incorrect or missing units, the final (A1) is not awarded.

(a) $200-190(0.97)^0$	(M1)	
Note: Award (M1) for correct substitution.		
=10 °C	(A1)	(C2)
Note: Units are required.		
(b) $200-190(0.97)^{30}$	(M1)	
Note: Award (M1) for correct substitution.		
=124 °C (123.808 °C)	(A1)	(C2)
<b>Note:</b> Units are required, unless already omitted in part (a).		
(c) $200 - 190(0.97)^k = 40$	(M1)	
Note: Award (M1) for correct substitution.		
k = 5.64 (minutes) (5.64198)	(A1)	(C2)
	[6	ó marks]



15. (a) 
$$2x - \frac{a}{x^2}$$
 (A1)(A1)(A1) (C3)  
Notes: Award (A1) for 2x, (A1) for -a and (A1) for  $x^{-2}$ .  
Award at most (A1)(A1)(A0) if extra terms are present.  
(b)  $2(1) - \frac{a}{1^2} = -14$  (M1)(M1)  
Note: Award (M1) for substituting 1 into their gradient function,  
(M1) for equating their gradient function to  $-14$ .  
Award (M0)(M0)(A0) if the original function is used instead  
of the gradient function.  
 $a = 16$  (A1)(ft) (C3)  
Note: Follow through from their gradient function from part (a).  
[6 marks]