Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2019

Mathematics

Higher level

Paper 1

17 pages

Section A

1. attempting to form two equations involving u_1 and d ***M1***

$(u_1+2d)+(u_1+7d)=1$ and $\frac{7}{2}[2u_1+6d]=35$

$2u_1+9d=1$

$14u_1+42d=70$ $(2u_1+6d=10)$ ***A1***

Note: Award ***A1*** for any two correct equations

attempting to solve their equations: ***M1***

$u_1=14$, $d=-3$ ***A1***

[4 marks]

2. (a) (i) $\overrightarrow{AB}=\begin{pmatrix}0\\2\\-2\end{pmatrix}$ ***A1***

(ii) $\overrightarrow{AC}=\begin{pmatrix}3\\1\\-2\end{pmatrix}$ ***A1***

Note: Accept row vectors or equivalent.

[2 marks]

(b) **METHOD 1**

attempt at vector product using $\overrightarrow{AB}$ and $\overrightarrow{AC}$. ***(M1)***

$\pm(2\boldsymbol{i}+6\boldsymbol{j}+6\boldsymbol{k})$ ***A1***

attempt to use area = $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$ ***M1***

$=\frac{\sqrt{76}}{2}\left(=\sqrt{19}\right)$ ***A1***

[4 marks]

continued…

Question 2 continued

METHOD 2

attempt to use $\vec{AB} \bullet \vec{AC} = \left|\vec{AB}\right|\left|\vec{AC}\right|\cos\theta$ ***M1***

$$\begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = \sqrt{0^2 + 2^2 + (-2)^2}\sqrt{3^2 + 1^2 + (-2)^2}\cos\theta$$

$6 = \sqrt{8}\sqrt{14}\cos\theta$ ***A1***

$$\cos\theta = \frac{6}{\sqrt{8}\sqrt{14}} = \frac{6}{\sqrt{112}}$$

attempt to use area $= \frac{1}{2}\left|\vec{AB}\right|\left|\vec{AC}\right|\sin\theta$ ***M1***

$$= \frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{1 - \frac{36}{112}}\left(= \frac{1}{2}\sqrt{8}\sqrt{14}\sqrt{\frac{76}{112}}\right)$$

$= \frac{\sqrt{76}}{2}\left(= \sqrt{19}\right)$ ***A1***

[4 marks]

Total [6 marks]

3. $g(x) = f(x+2)\left(= (x+2)^4 - 6(x+2)^2 - 2(x+2) + 4\right)$ ***M1***

attempt to expand $(x+2)^4$ ***M1***

$(x+2)^4 = x^4 + 4(2x^3) + 6(2^2x^2) + 4(2^3x) + 2^4$ ***(A1)***

$= x^4 + 8x^3 + 24x^2 + 32x + 16$ ***A1***

$g(x) = x^4 + 8x^3 + 24x^2 + 32x + 16 - 6(x^2 + 4x + 4) - 2x - 4 + 4$

$= x^4 + 8x^3 + 18x^2 + 6x - 8$ ***A1***

Note: For correct expansion of $f(x-2) = x^4 - 8x^3 + 18x^2 - 10x$ award max ***M0M1(A1)A0A1***.

[5 marks]

4. $u = \sin x \Rightarrow \mathrm{d}u = \cos x\mathrm{d}x$ ***(A1)***

valid attempt to write integral in terms of u and $\mathrm{d}u$ ***M1***

$\int \frac{\cos^3 x \ \mathrm{d}x}{\sqrt{\sin x}} = \int \frac{\left(1-u^2\right) \ \mathrm{d}u}{\sqrt{u}}$ ***A1***

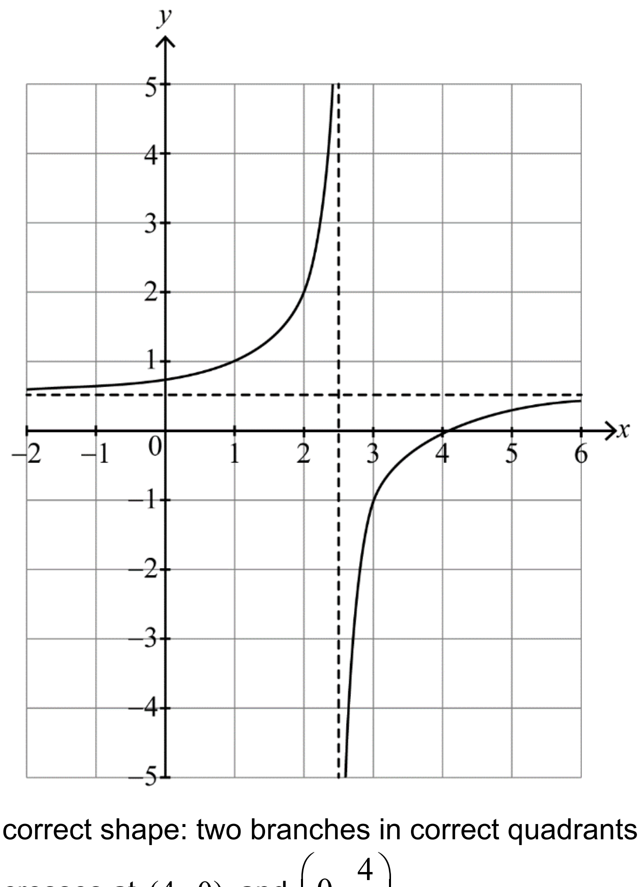
$= \int \left(u^{-\frac{1}{2}} - u^{\frac{3}{2}}\right) \mathrm{d}u$

$= 2u^{\frac{1}{2}} - \frac{2u^{\frac{5}{2}}}{5} (+c)$ ***(A1)***

$= 2\sqrt{\sin x} - \frac{2\left(\sqrt{\sin x}\right)^5}{5} (+c)$ or equivalent ***A1***

[5 marks]

5. (a)



correct shape: two branches in correct quadrants with asymptotic behaviour ***A1***

crosses at $(4, 0)$ and $\left(0, \frac{4}{5}\right)$ ***A1A1***

asymptotes at $x = \frac{5}{2}$ and $y = \frac{1}{2}$ ***A1A1***

[5 marks]

continued…

Question 5 continued

(b) (i) $x < \frac{5}{2}, x \geq 4$ ***A1A1***

(ii) $f(x) \geq 0, f(x) \neq \frac{1}{\sqrt{2}} \left(f(x) \in \mathbb{R} \right)$ ***A1***

Note: Follow through from their graph, as long as it is a rectangular hyperbola.

Note: Allow range expressed in terms of y.

[3 marks]

Total [8 marks]

6. (a) attempt to differentiate implicitly ***M1***

$\frac{dy}{dx} = x\sec^2\left(\frac{\pi xy}{4}\right)\left[\frac{\pi}{4}x\frac{dy}{dx} + \frac{\pi}{4}y\right] + \tan\left(\frac{\pi xy}{4}\right)$ ***A1A1***

Note: Award ***A1*** for each term.

attempt to substitute $x=1,\ y=1$ into their equation for $\frac{dy}{dx}$ ***M1***

$\frac{dy}{dx} = \frac{\pi}{2}\frac{dy}{dx} + \frac{\pi}{2} + 1$

$\frac{dy}{dx}\left(1 - \frac{\pi}{2}\right) = \frac{\pi}{2} + 1$ ***A1***

$\frac{dy}{dx} = \frac{2+\pi}{2-\pi}$ ***AG***

[5 marks]

(b) attempt to use gradient of normal $= \frac{-1}{\frac{dy}{dx}}$ ***(M1)***

$= \frac{\pi - 2}{\pi + 2}$

so equation of normal is $y - 1 = \frac{\pi - 2}{\pi + 2}(x - 1)$ or $y = \frac{\pi - 2}{\pi + 2}x + \frac{4}{\pi + 2}$ ***A1***

[2 marks]

Total [7 marks]

7. use of at least one "log rule" applied correctly for the first equation ***M1***

$\log_2 6x = \log_2 2 + 2\log_2 y$

$= \log_2 2 + \log_2 y^2$

$= \log_2 \left(2y^2\right)$

$\Rightarrow 6x = 2y^2$ ***A1***

use of at least one "log rule" applied correctly for the second equation ***M1***

$\log_6 \left(15y - 25\right) = 1 + \log_6 x$

$= \log_6 6 + \log_6 x$

$= \log_6 6x$

$\Rightarrow 15y - 25 = 6x$ ***A1***

attempt to eliminate x (or y) from their two equations ***M1***

$2y^2 = 15y - 25$

$2y^2 - 15y + 25 = 0$

$\left(2y - 5\right)\left(y - 5\right) = 0$

$x = \frac{25}{12}, \; y = \frac{5}{2},$ ***A1***

or $x = \frac{25}{3}, \; y = 5$ ***A1***

Note: x, y values do not have to be "paired" to gain either of the final two ***A*** marks.

[7 marks]

8. (a) attempt to use Pythagoras in triangle OXB ***M1***

$\Rightarrow r^2 = R^2 - \left(h - R\right)^2$ ***A1***

substitution of their r^2 into formula for volume of cone $V = \frac{\pi r^2 h}{3}$ ***M1***

$= \frac{\pi h}{3}\left(R^2 - \left(h - R\right)^2\right)$

$= \frac{\pi h}{3}\left(R^2 - \left(h^2 + R^2 - 2hR\right)\right)$ ***A1***

Note: This **A** mark is independent and may be seen anywhere for the correct expansion of $\left(h - R\right)^2$.

$= \frac{\pi h}{3}\left(2hR - h^2\right)$

$= \frac{\pi}{3}\left(2Rh^2 - h^3\right)$ ***AG***

[4 marks]

continued…

Question 8 continued

(b) at max, $\frac{\mathrm{d}V}{\mathrm{d}h}=0$ ***R1***

$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\pi}{3}\left(4Rh-3h^2\right)$

$\Rightarrow 4Rh=3h^2$

$\Rightarrow h=\frac{4R}{3}$ (since $h\neq 0$) ***A1***

EITHER

$V_{\max}=\frac{\pi}{3}\left(2Rh^2-h^3\right)$ from part (a)

$=\frac{\pi}{3}\left(2R\left(\frac{4R}{3}\right)^2-\left(\frac{4R}{3}\right)^3\right)$ ***A1***

$=\frac{\pi}{3}\left(2R\frac{16R^2}{9}-\left(\frac{64R^3}{27}\right)\right)$ ***A1***

OR

$r^2=R^2-\left(\frac{4R}{3}-R\right)^2$

$r^2=R^2-\frac{R^2}{9}=\frac{8R^2}{9}$ ***A1***

$\Rightarrow V_{\max}=\frac{\pi r^2}{3}\left(\frac{4R}{3}\right)$

$=\frac{4\pi R}{9}\left(\frac{8R^2}{9}\right)$ ***A1***

THEN

$=\frac{32\pi R^3}{81}$ ***AG***

[4 marks]

Total [8 marks]

Section B

9. (a) $3\cos 2x = 4 - 11\cos x$

attempt to form a quadratic in $\cos x$ ***M1***

$3\left(2\cos^2 x - 1\right) = 4 - 11\cos x$ ***A1***

$\left(6\cos^2 x + 11\cos x - 7 = 0\right)$

valid attempt to solve their quadratic ***M1***

$(3\cos x + 7)(2\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ ***A1***

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ ***A1A1***

Note: Ignore any "extra" solutions.

[6 marks]

(b) consider $(\pm)\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (4 - 11\cos x - 3\cos 2x)\,\mathrm{d}x$ ***M1***

$= (\pm)\left[4x - 11\sin x - \frac{3}{2}\sin 2x\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$ ***A1***

Note: Ignore lack of or incorrect limits at this stage.

attempt to substitute their limits into their integral ***M1***

$= \frac{20\pi}{3} - 11\sin\frac{5\pi}{3} - \frac{3}{2}\sin\frac{10\pi}{3} - \left(\frac{4\pi}{3} - 11\sin\frac{\pi}{3} - \frac{3}{2}\sin\frac{2\pi}{3}\right)$

$= \frac{16\pi}{3} + \frac{11\sqrt{3}}{2} + \frac{3\sqrt{3}}{4} + \frac{11\sqrt{3}}{2} + \frac{3\sqrt{3}}{4}$

$= \frac{16\pi}{3} + \frac{25\sqrt{3}}{2}$ ***A1A1***

[5 marks]

(c) attempt to differentiate both functions and equate ***M1***

$-6\sin 2x = 11\sin x$ ***A1***

attempt to solve for x ***M1***

$11\sin x + 12\sin x\cos x = 0$

$\sin x(11 + 12\cos x) = 0$

$\cos x = -\frac{11}{12}$ $\left(\text{or } \sin x = 0\right)$ ***A1***

$\Rightarrow y = 4 - 11\left(-\frac{11}{12}\right)$ ***M1***

$y = \frac{169}{12}\left(= 14\frac{1}{12}\right)$ ***A1***

[6 marks]

Total [17 marks]

10. (a) mode is 0 ***A1***

[1 mark]

(b) (i) attempt at integration by parts ***(M1)***

$\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{\sqrt{1-x^2}}, \mathrm{d}v=\mathrm{d}x$

$=x\arcsin x-\int\frac{x\mathrm{d}x}{\sqrt{1-x^2}}$ ***A1***

$=x\arcsin x+\sqrt{1-x^2}\;(+c)$ ***A1***

(ii) $\int_0^1(\pi-\arcsin x)\;\mathrm{d}x=\left[\pi x-x\arcsin x-\sqrt{1-x^2}\right]_0^1$ ***A1***

$=\left(\pi-\frac{\pi}{2}-0\right)-(0-0-1)=\frac{\pi}{2}+1$

$=\frac{\pi+2}{2}$ ***A1***

$\int_0^1 k(\pi-\arcsin x)\;\mathrm{d}x=1$ ***(M1)***

Note: This line can be seen (or implied) anywhere.

Note: Do not allow ***FT A*** marks from bi to bii.

$k\left(\frac{\pi+2}{2}\right)=1$

$\Rightarrow k=\frac{2}{2+\pi}$ ***AG***

[6 marks]

(c) (i) attempt to use product rule to differentiate ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}x}=x\arcsin x+\frac{x^2}{2\sqrt{1-x^2}}-\frac{1}{4\sqrt{1-x^2}}-\frac{x^2}{4\sqrt{1-x^2}}+\frac{\sqrt{1-x^2}}{4}$ ***A2***

Note: Award ***A2*** for all terms correct, ***A1*** for 4 correct terms.

$=x\arcsin x+\frac{2x^2}{4\sqrt{1-x^2}}-\frac{1}{4\sqrt{1-x^2}}-\frac{x^2}{4\sqrt{1-x^2}}+\frac{1-x^2}{4\sqrt{1-x^2}}$ ***A1***

Note: Award ***A1*** for equivalent combination of correct terms over a common denominator.

$=x\arcsin x$ ***AG***

continued…

Question 10 continued

(ii) $\mathrm{E}(X) = k\int_0^1 x(\pi - \arcsin x)\,\mathrm{d}x$ ***M1***

$= k\int_0^1 (\pi x - x\arcsin x)\,\mathrm{d}x$

$= k\left[\frac{\pi x^2}{2} - \frac{x^2}{2}\arcsin x + \frac{1}{4}\arcsin x - \frac{x}{4}\sqrt{1-x^2}\right]_0^1$ ***A1A1***

Note: Award ***A1*** for first term, ***A1*** for next 3 terms.

$= k\left[\left(\frac{\pi}{2} - \frac{\pi}{4} + \frac{\pi}{8}\right) - (0)\right]$ ***A1***

$= \left(\frac{2}{2+\pi}\right)\frac{3\pi}{8}$ ***A1***

$= \frac{3\pi}{4(\pi+2)}$ ***AG***

[9 marks]

Total [16 marks]

11. (a) translation k units to the left (or equivalent) ***A1***

[1 mark]

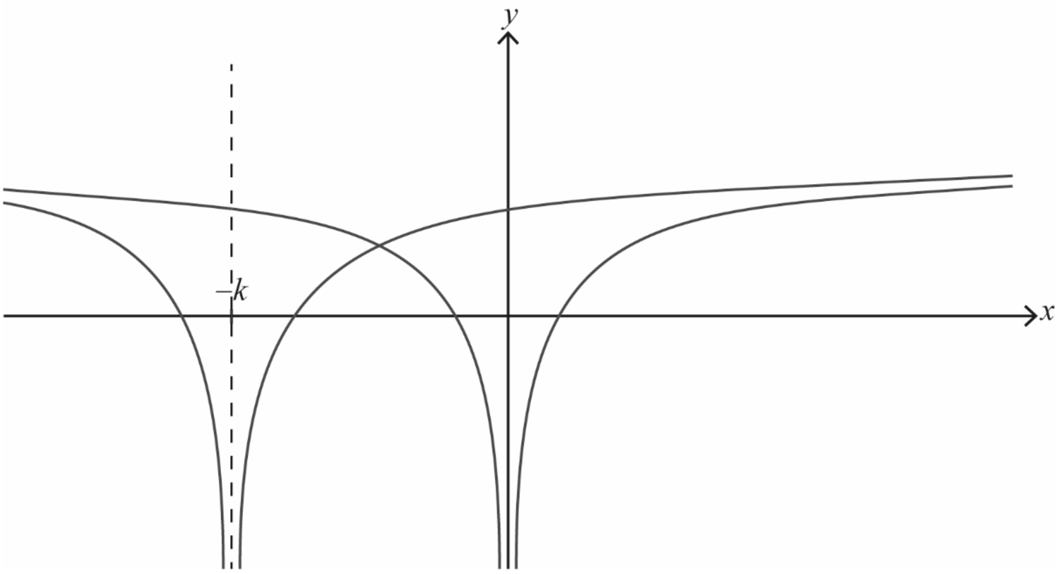
(b) range is $(g(x) \in)\mathbb{R}$ ***A1***

[1 mark]

continued…

Question 11 continued

(c)



correct shape of $y = f(x)$ **A1**

their $f(x)$ translated k units to left (possibly shown by $x = -k$ marked on x-axis) **A1**

asymptote included and marked as $x = -k$ **A1**

$f(x)$ intersects x-axis at $x = -1,\ x = 1$ **A1**

$g(x)$ intersects x-axis at $x = -k - 1,\ x = -k + 1$ **A1**

$g(x)$ intersects y-axis at $y = \ln k$ **A1**

Note: Do not penalise candidates if their graphs "cross" as $x \to \pm\infty$.

Note: Do not award ***FT*** marks from the candidate's part (a) to part (c).

[6 marks]

(d) at P $\ln(x+k) = \ln(-x)$

attempt to solve $x + k = -x$ (or equivalent) ***(M1)***

$x = -\frac{k}{2} \Rightarrow y = \ln\left(\frac{k}{2}\right)$ (or $y = \ln\left|\frac{k}{2}\right|$) **A1**

$\text{P}\left(-\frac{k}{2}, \ln\frac{k}{2}\right)$ (or $\text{P}\left(-\frac{k}{2}, \ln\left|\frac{k}{2}\right|\right)$)

[2 marks]

continued…

Question 11 continued

(e) attempt to differentiate $\ln(-x)$ or $\ln|x|$ ***(M1)***

$\frac{dy}{dx} = \frac{1}{x}$ ***A1***

at P, $\frac{dy}{dx} = \frac{-2}{k}$ ***A1***

recognition that tangent passes through origin $\Rightarrow \frac{y}{x} = \frac{dy}{dx}$ ***(M1)***

$\frac{\ln\left(\frac{k}{2}\right)}{-\frac{k}{2}} = \frac{-2}{k}$ ***A1***

$\ln\left(\frac{k}{2}\right) = 1$ ***(A1)***

$\Rightarrow k = 2e$ ***A1***

[7 marks]

Note:	For candidates who explicitly differentiate $\ln(x)$ (rather than $\ln(-x)$ or $\ln|x|$, award ***M0A0A1M1A1A1A1***.

Total [17 marks]