Normal distribution 03.02 *[106 marks]*

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

1a. Find the probability that a bag selected at random is rejected. *[2 marks]*

.....

.....

.....

.....

.....

.....

1b. Estimate the number of bags which will be rejected from a random sample of 100 bags. *[1 mark]*

.....

.....

.....

1c. Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. *[3 marks]*

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean 102 g and standard deviation 8 g.

2a. Find the probability that a randomly selected packet has a weight less than 100 g. *[2 marks]*

2b. The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. *[2 marks]*

2c. A packet is randomly selected. Given that the packet has a weight greater than 105 g, find the probability that it has a weight greater than 110 g. *[3 marks]*

2d. From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. *[3 marks]*

2e. Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than 95 g. *[4 marks]*

3. Rachel and Sophia are competing in a javelin-throwing competition. *[7 marks]*

The distances, R metres, thrown by Rachel can be modelled by a normal distribution with mean 56.5 and standard deviation 3.

The distances, S metres, thrown by Sophia can be modelled by a normal distribution with mean 57.5 and standard deviation 1.8.

In the first round of competition, each competitor must have five throws. To qualify for the next round of competition, a competitor must record at least one throw of 60 metres or greater in the first round.

Find the probability that only one of Rachel or Sophia qualifies for the next round of competition.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The length, X mm, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

4a. Find P($24.15 < X < 25$). *[2 marks]*

4b. Find σ, the standard deviation of X. *[3 marks]*

4c. Hence, find the probability that a seashell selected at random has a length greater than 26 mm. *[2 marks]*

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

4d. Find E(Y). *[3 marks]*

4e. Find the probability that exactly three of these seashells have a length greater than 26 mm. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4f. A seashell selected at random has a length less than 26 mm. *[3 marks]*

Find the probability that its length is between 24.15 mm and 25 mm.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The time, T minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

5a. By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. *[4 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Use $\mu = 32.29$ in the remainder of the question.

5b. Find the 86th percentile time to complete the jigsaw puzzle. *[2 marks]*

5c. Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. *[2 marks]*

Six randomly chosen people complete the jigsaw puzzle.

5d. Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5e. Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle. *[4 marks]*

Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

6a. Find the value of σ. *[4 marks]*

6b. On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. *[2 marks]*

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

6c. Find the probability that she will be late to work at least one day next week. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6d. Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. *[5 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

6e. Find the probability that Suzi will receive a bonus. *[4 marks]*

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

7a. Find the probability that a randomly selected chocolate muffin weighs less than 61 g. *[2 marks]*

7b. In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

7c. Find the probability that the randomly selected muffin weighs less than 61 g. *[4 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

.....

7d. Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

7e. Find the value of σ. *[5 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The flight times, T minutes, between two cities can be modelled by a normal distribution with a mean of 75 minutes and a standard deviation of σ minutes.

8a. Given that 2% of the flight times are longer than 82 minutes, find the value of σ. *[3 marks]*

8b. Find the probability that a randomly selected flight will have a flight time of more than 80 minutes. *[2 marks]*

8c. Given that a flight between the two cities takes longer than 80 minutes, find the probability that it takes less than 82 minutes. *[4 marks]*

On a particular day, there are 64 flights scheduled between these two cities.

8d. Find the expected number of flights that will have a flight time of more than 80 minutes. *[3 marks]*

8e. Find the probability that more than 6 of the flights on this particular day will have a flight time of more than 80 minutes. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .