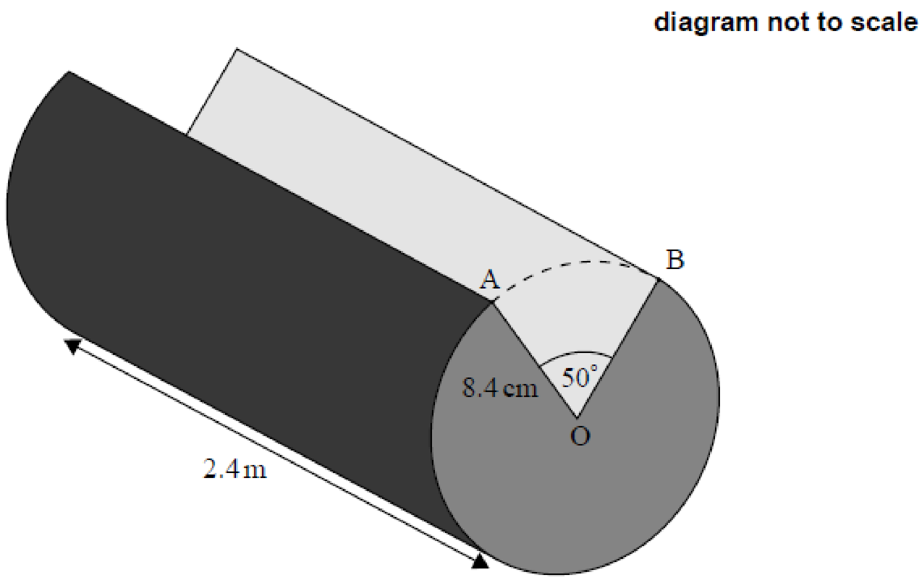
Review Set 1 [91 marks]

1. [Maximum mark: 4] SPM.1.SL.TZ0.11

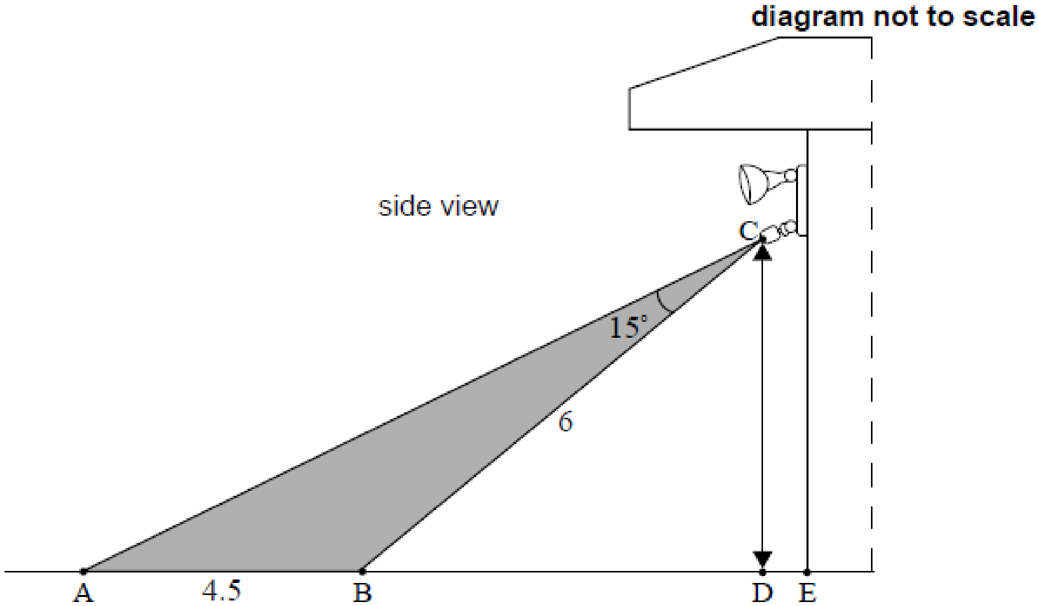
Helen is building a cabin using cylindrical logs of length 2.4 m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.



Find the volume of this log. [4]

2. [Maximum mark: 8] SPM.1.SL.TZ0.14

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $A\hat{C}B$ is 15°.



(a) Find CÂB. [3]

(b) Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

3. [Maximum mark: 12] SPM.2.SL.TZ0.5

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s\text{ m s}^{-1}$, and braking distance, $d\text{ m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s\text{ m s}^{-1}$	0	6	10
Braking distance, $d\text{ m}$	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where $p, q \in \mathbb{Z}$

At a speed of $6\ \mathrm{m\,s^{-1}}$, Model A can be represented by the equation $6p + q = 2$.

(a.i) Write down a second equation to represent Model A, when the speed is $10\ \mathrm{m\,s^{-1}}$. [2]

(a.ii) Find the values of p and q. [2]

(a.iii) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(a.iiii) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \le s \le 10$ and $-10 \le d \le 60$, clearly showing the vertex. [3]

(a.iiiii) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d(s) = 0.95s^2 - 3.92s$

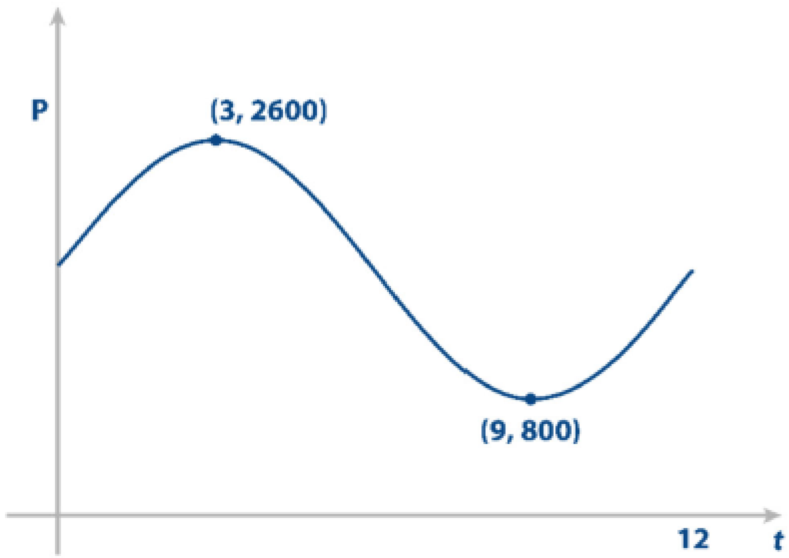
(a.iiiiii) Use Model B to calculate an estimate for the braking distance at a speed of $20\ \mathrm{m\,s^{-1}}$. [2]

4. [Maximum mark: 7] EXN.1.SL.TZ0.6

The size of the population (P) of migrating birds in a particular town can be approximately modelled by the equation $P = a\sin(bt) + c,\ \ a, b, c \in \mathbb{R}^+$, where t is measured in months from the time of the initial measurements.

In a 12 month period the maximum population is 2600 and occurs when $t = 3$ and the minimum population is 800 and occurs when $t = 9$.

This information is shown on the graph below.



(a.i) Find the value of a. [2]

(a.ii) Find the value of b. [2]

(a.iii) Find the value of c. [1]

(b) Find the value of t at which the population first reaches 2200. [2]

5. [Maximum mark: 9] EXN.1.SL.TZ0.11

A farmer owns a triangular field ABC. The length of side $[\mathrm{AB}]$ is $85\,\mathrm{m}$ and side $[\mathrm{AC}]$ is $110\,\mathrm{m}$. The angle between these two sides is 55°.

(a) Find the area of the field. [3]

(b) The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on $[\mathrm{BC}]$.

Find BD. Fully justify any assumptions you make.

[6]

6. [Maximum mark: 13] EXM.2.SL.TZ0.3

Urvashi wants to model the height of a moving object. She collects the following data showing the height, h metres, of the object at time t seconds.

t (seconds)	2	5	7
h (metres)	34	38	24

She believes the height can be modeled by a quadratic function, $h\left(t\right)=at^{2}+bt+c$, where $a,\ b,\ c\in\mathbb{R}$.

(a) Show that $4a+2b+c=34$. [1]

(b) Write down two more equations for a, b and c. [3]

(c) Solve this system of three equations to find the value of a, b and c. [4]

Hence find

(d.i) when the height of the object is zero. [3]

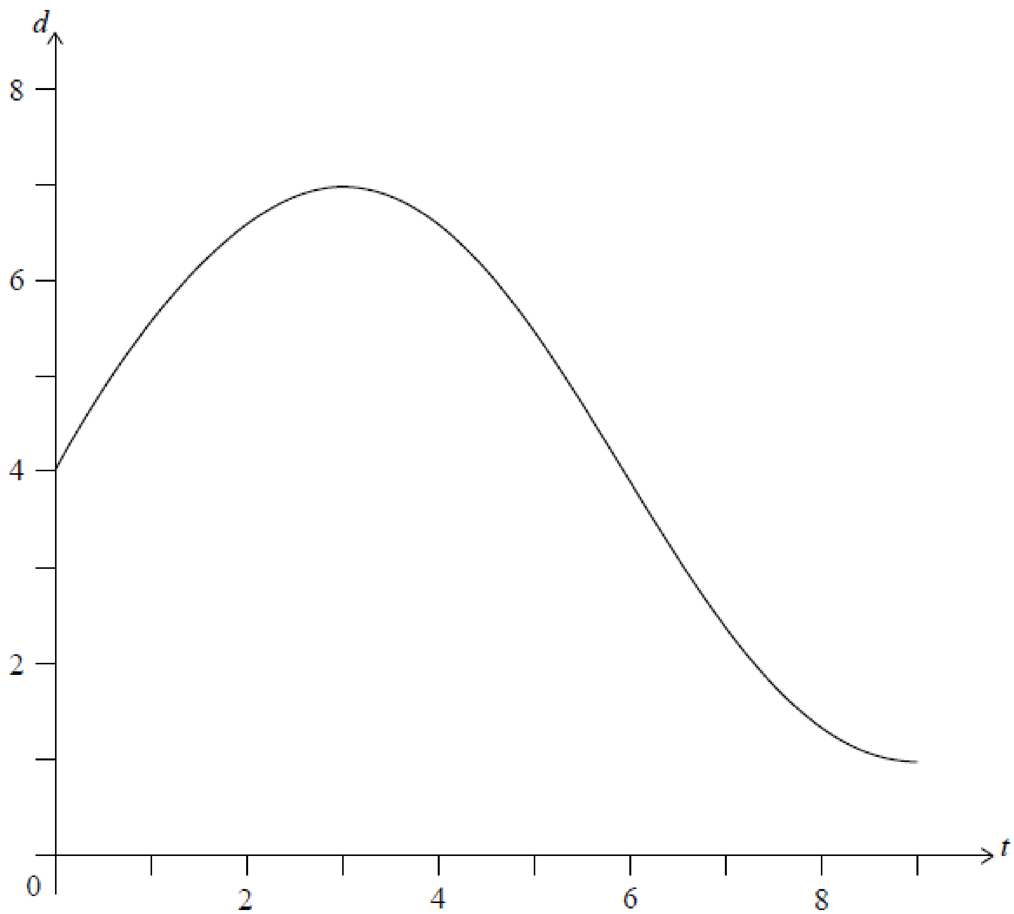
(d.ii) the maximum height of the object. [2]

7. [Maximum mark: 6] 24M.1.SL.TZ1.7

The following graph shows the depth of water, d metres, in a river at t hours after $12:00$.

At $15:00$, the depth of water reaches $7\,\mathrm{m}$, its highest level. At $21:00$, the depth of water drops to $1\,\mathrm{m}$, its lowest level.

The depth can be modelled by the function $d(t) = a\sin(bt) + 4$.



(a) Find the value of a. [1]

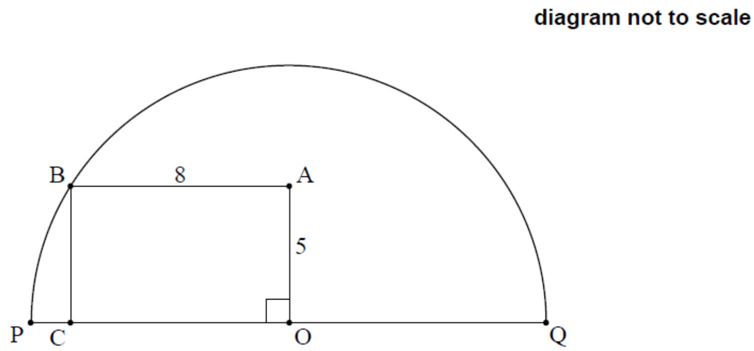
(b) Find the value of b. [2]

(c) Find the first time after $12:00$ when the depth is equal to $3\,\mathrm{m}$. Give your answer to the nearest minute. [3]

8. [Maximum mark: 5] 24M.1.SL.TZ1.11

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle OABC is also shown, such that $\mathrm{AB} = 8$ and

$\mathrm{OA} = 5$.

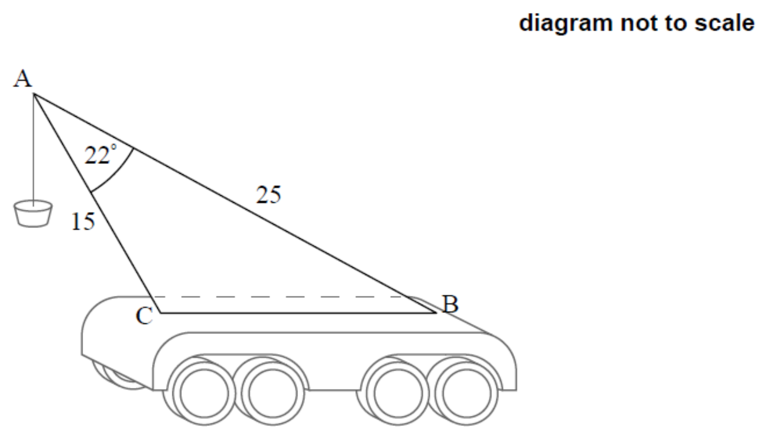


[5]

Find the length of the arc BQ.

9. [Maximum mark: 6] 24M.1.SL.TZ2.2

The diagram shows a toy crane.



$\mathrm{AB} = 25\ \mathrm{cm}$, $\mathrm{AC} = 15\ \mathrm{cm}$ and $\mathrm{B\hat{A}C} = 22^{\circ}$.

(a) Calculate BC. [3]

(b) Given that $\mathrm{A\hat{B}C}$ is acute, calculate $\mathrm{A\hat{B}C}$ [3]

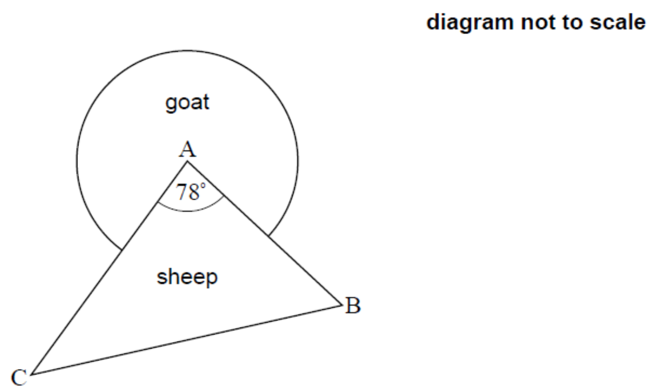
10. [Maximum mark: 6] 24M.1.SL.TZ2.11

A sheep is in a field in the shape of a triangle, ABC.

$\mathrm{AC} = 21$ metres, $\mathrm{AB} = 15$ metres and $\mathrm{C\hat{A}B} = 78^{\circ}$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

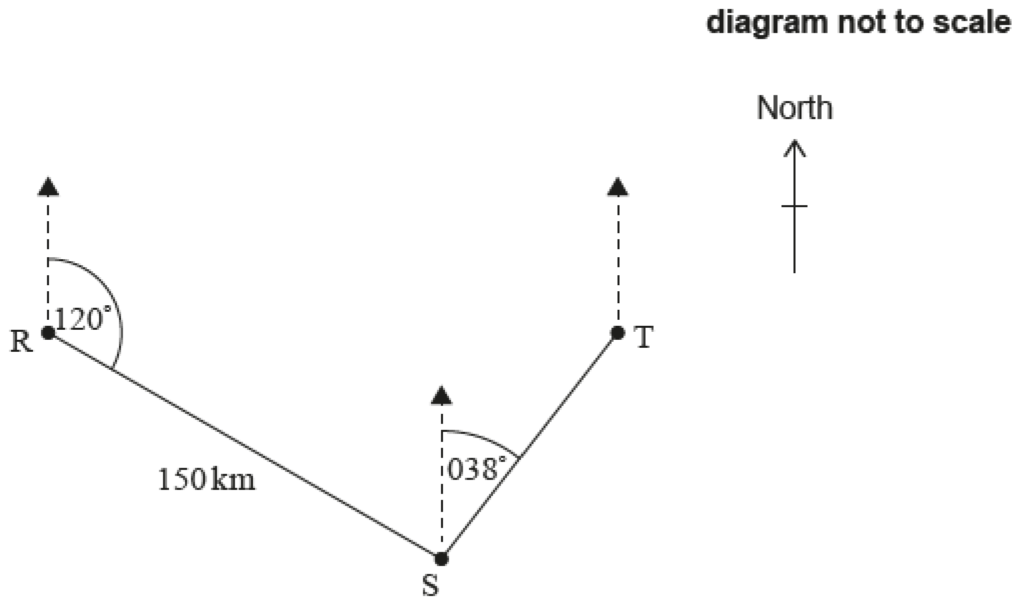
The fields are shown in the diagram.



Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field. [6]

11. [Maximum mark: 6] 23N.1.SL.TZ1.5

Ron sails his boat from point R for a distance of $150\ \mathrm{km}$, on a bearing of 120°, to arrive at point S. He then sails on a bearing of 038° to point T. Ron's journey is shown in the diagram.



(a) Find $\mathrm{R\hat{S}T}$. [2]

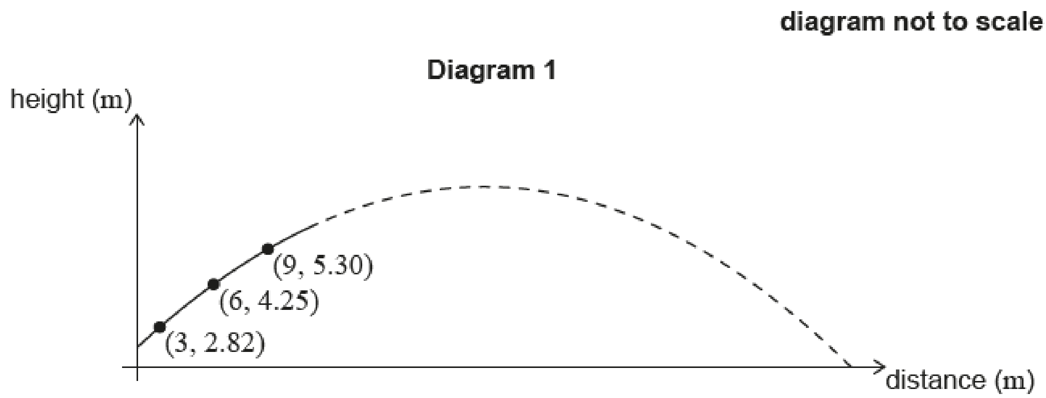
Point T is directly east of point R.

(b) Calculate the distance that Ron sails to return directly from point T to point R. [4]

12. [Maximum mark: 9] 23N.1.SL.TZ1.7

An athlete on a horizontal athletic field throws a discus. The height of the discus above the field, in metres, after it is thrown can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the discus has travelled from the athlete.

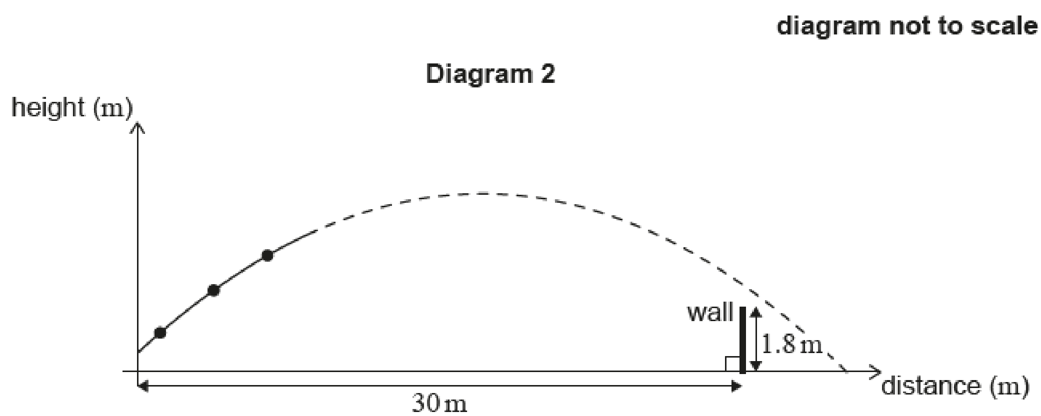
A specialized camera tracks the initial path of the discus after it is thrown by the athlete. The camera records that the discus travels through the three points $(3\ ,\ 2.82)$, $(6\ ,\ 4.25)$ and $(9\ ,\ 5.30)$, as shown in **Diagram 1**.



(a) Use the coordinates $(3\ ,\ 2.82)$ to write down an equation in terms of a, b and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the discus. [3]

A 1.8-metre-high wall is 30 metres from where the athlete threw the discus, as shown in **Diagram 2**.



(c) Show that the model predicts that the discus will go over the wall. [3]

(d) Find the horizontal distance that the discus will travel, from the athlete until it first hits the ground, according to this model. [2]