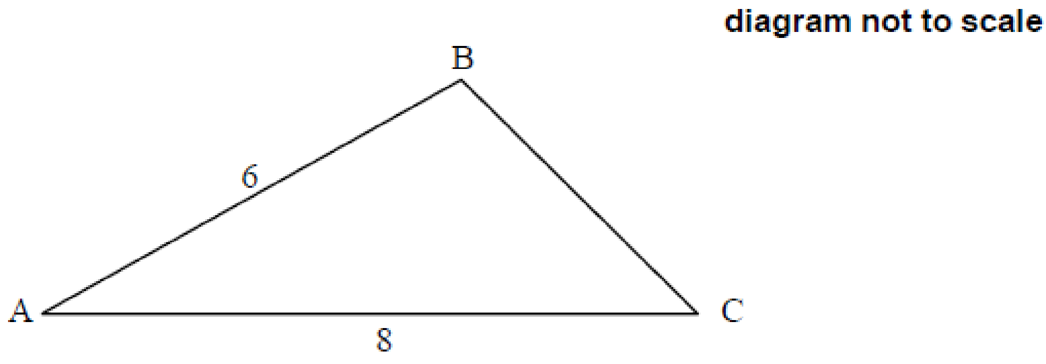
Review Set 2 [75 marks]

1. [Maximum mark: 5] SPM.1.SL.TZ0.1

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



(a) Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. [3]

Markscheme

valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

(b) Find the area of triangle ABC. [2]

Markscheme

$\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

area $= 4\sqrt{11}$ ***A1***

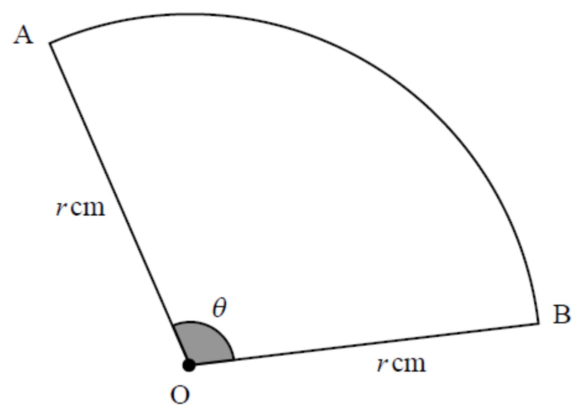
[2 marks]

2. [Maximum mark: 8] 24M.1.SL.TZ1.4

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\ \mathrm{cm}$ and the area of the sector is $6.25\ \mathrm{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

Markscheme

$2r + r\theta = 10$ ***A1***

$\frac{1}{2}r^2\theta = 6.25$ ***A1***

attempt to eliminate θ to obtain an equation in r ***M1***

correct intermediate equation in r ***A1***

$10-2r=\frac{25}{2r}$ OR $\frac{10}{r}-2=\frac{25}{2r^2}$ OR $\frac{1}{2}r^2\left(\frac{10}{r}-2\right)=6.25$ OR $1.25+2r^2=10r$

$4r^2-20r+25=0$ ***AG***

[4 marks]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

Markscheme

attempt to solve quadratic by factorizing or use of formula or completing the square ***(M1)***

$(2r-5)^2=0$ OR $r=\frac{20\pm\sqrt{(-20)^2-4(4)(25)}}{2(4)}\left(=\frac{20\pm\sqrt{400-400}}{8}\right)$

$r=\frac{5}{2}$ ***A1***

attempt to substitute their value of r into their perimeter or area equation ***(M1)***

$\theta=\frac{10-2\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)}$ or $\theta=\frac{25}{2\left(\frac{5}{2}\right)^2}$

$\theta=2$ ***A1***

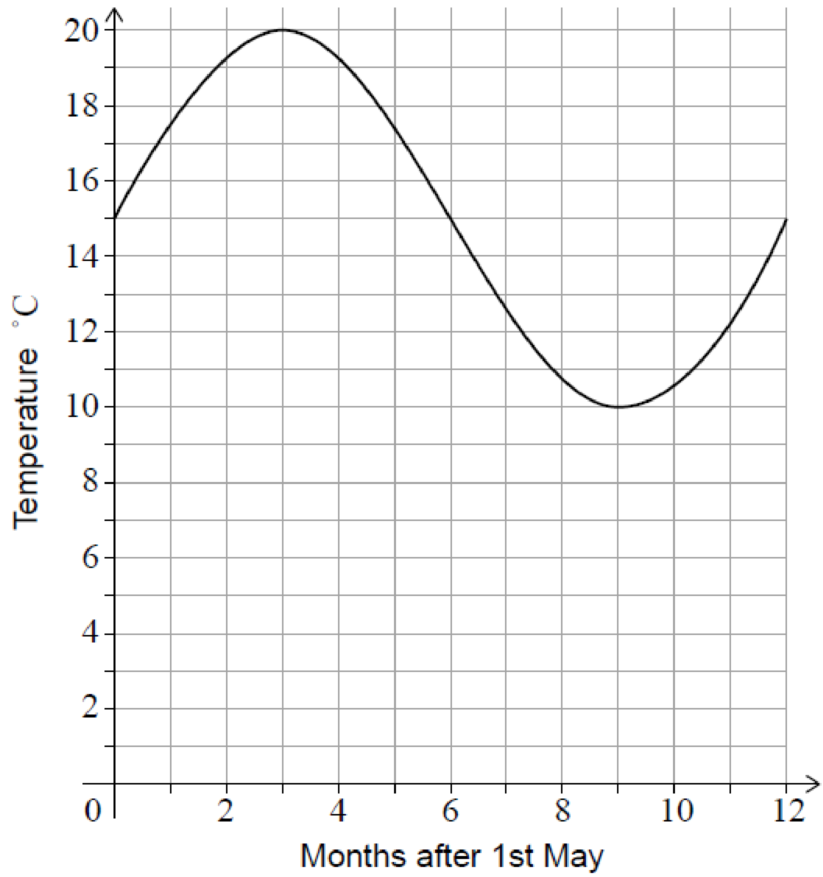
[4 marks]

3. [Maximum mark: 12] 24M.1.SL.TZ2.7

Alex only swims in the sea if the water temperature is at least $15\,^{\circ}\mathrm{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a \sin bx + c$ for $0 \le x \le 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(a) Show that $b = \frac{\pi}{6}$. [1]

Markscheme

$12b = 2\pi$ OR $(b =)\frac{2\pi}{12}$ OR $12 = \frac{2\pi}{b}$ ***A1***

$b = \frac{\pi}{6}$ ***AG***

[1 mark]

(b) Write down the value of

(b.i) a; [1]

Markscheme

$a = 5$ ***A1***

[1 mark]

(b.ii) c. [1]

Markscheme

$c = 15$ ***A1***

[1 mark]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5 \sin \frac{\pi}{6}x + 11$, where $0 \leq x \leq 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(c.i) find the midday water temperature on 1st October, five months after 1st May. [3]

Markscheme

attempt to substitute $x = 5$ into $g(x)$ ***(M1)***

$g(5) = 3.5 \sin \frac{5\pi}{6} + 11$

$\sin \frac{5\pi}{6} = \frac{1}{2}$ ***(A1)***

$g(5) = 3.5 \times \frac{1}{2} + 11$

$g(5) = 12.75 \left(= \frac{51}{4}\right)$ ***A1***

[3 marks]

(c.ii) show that the midday water temperature is never warm enough for Alex to swim. [3]

Markscheme

METHOD 1 (finding maximum temperature)

considering the maximum value of $\sin \frac{\pi}{6}x (= 1)$ OR $g\prime(x) = 0$ at maximum

OR maximum = vertical shift + amplitude (may be seen on a graph) ***(M1)***

$g_{\max} = 3.5 + 11$ OR $\frac{\pi}{6} \cdot 3.5 \cos \left(\frac{\pi}{6}x\right) = 0$ OR $x = 3$

$g_{\max} = 14.5$ ***A1***

$14.5 < 15$ (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

Note: Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***).

Worded conclusions are acceptable for the ***R1***, as long as the reasoning is clear that the water does not reach 15°, so not warm enough for Alex.

METHOD 2 (working with inequality)

$3.5 \sin \left(\frac{\pi}{6}x\right) + 11 \geq 15$ ***(M1)***

$\sin\left(\frac{\pi}{6}x\right) \geq \frac{8}{7}$ ***A1***

sine values can never be greater than 1 (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

Note: Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***).

If candidate works with an equation throughout, the ***M1*** and ***A1*** may be awarded, if appropriate. A correct inequality is required for the ***R1*** to be awarded.

[3 marks]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]

Markscheme

EITHER

attempt to find $0.7f(x)$ OR $0.7f(x) + q$ ***(M1)***

$0.7f(x) = 3.5 \sin \frac{\pi}{6}x + 10.5$ OR
$0.7f(x) + q = 3.5 \sin \frac{\pi}{6}x + 10.5 + q$ OR $10.5 + q = 11$
(A1)

OR

attempt to find $0.7f(x)$ for a particular value of x ***(M1)***

eg maximum $20 \times 0.7 = 14$ ***(A1)***

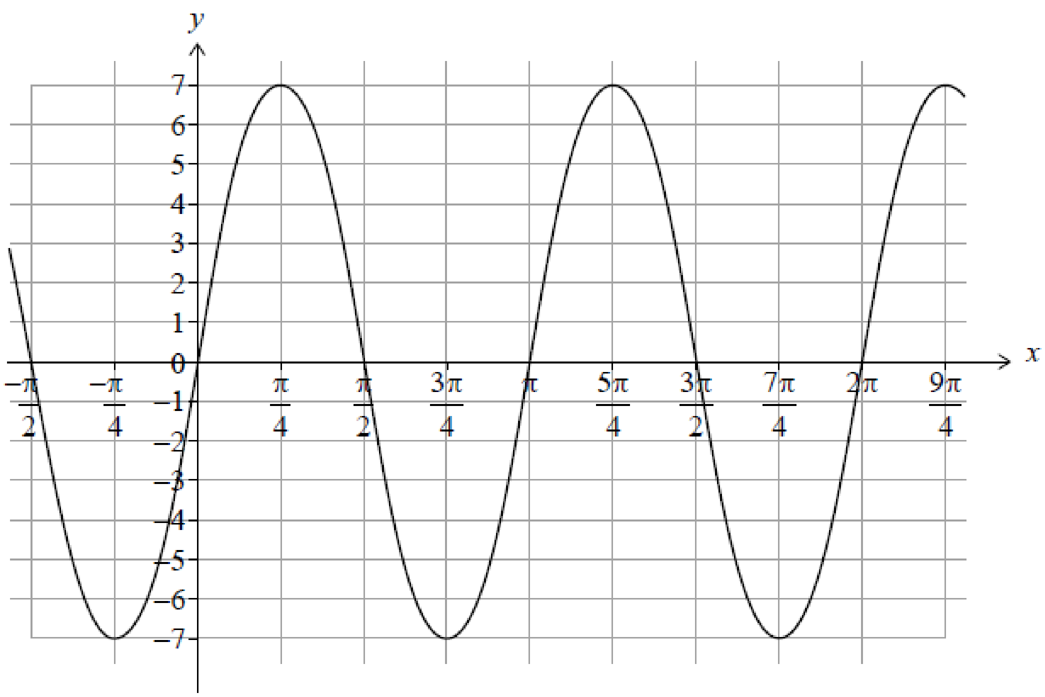
THEN

$q = 0.5$ ***A1***

[3 marks]

4. [Maximum mark: 7] 23N.1.SL.TZ1.1

Consider the function $f(x) = a \sin (bx)$ with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.



(a) Write down the value of a. [1]

Markscheme

$a = 7$ ***A1***

[1 mark]

(b.i) Write down the period of f. [1]

Markscheme

period $= \pi$ *A1*

[1 marks]

(b.ii) Hence, find the value of b. [2]

Markscheme

$b = \frac{2\pi}{\pi}$ OR $\pi = \frac{2\pi}{b}$ *(A1)*

$= 2$ *A1*

[2 marks]

(c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

Markscheme

substituting $\frac{\pi}{12}$ into their $f(x)$ *(M1)*

$f\left(\frac{\pi}{12}\right) = 7\sin\left(\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ *(A1)*

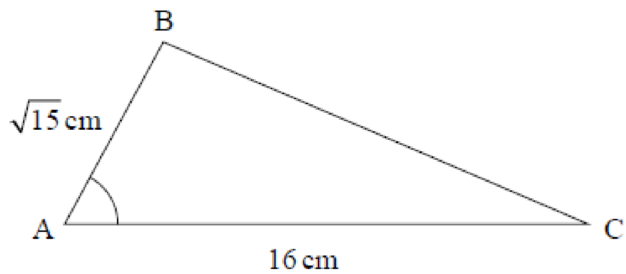
$= \frac{7}{2}$ **A1**

[3 marks]

5. [Maximum mark: 6] 23N.1.SL.TZ1.5

In the following triangle ABC, $\text{AB} = \sqrt{15 \text{ cm}}$, $\text{AC} = 16 \text{ cm}$ and $\cos \text{B}\hat{\text{A}}\text{C} = \frac{1}{4}$.

diagram not to scale



Find the area of triangle ABC. [6]

Markscheme

METHOD 1

EITHER

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\left(\sqrt{4^2 - 1^2} =\right) \sqrt{15}$ ***(A1)***

OR

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ ***(M1)***

$\sin^2 \text{B}\hat{\text{A}}\text{C} = 1 - \left(\frac{1}{4}\right)^2$ ***(A1)***

THEN

$\sin \text{B}\hat{\text{A}}\text{C} = \frac{\sqrt{15}}{4}$ (may be seen in area formula) ***A1*** attempt to use 'Area$=\frac{1}{2}ab\sin C$' (must include their calculated value of $\sin \mathrm{B\hat{A}C}$) *(M1)*

$=\frac{1}{2}\times 16\times\sqrt{15}\times\frac{\sqrt{15}}{4}$ *(A1)*

$=30\ (\mathrm{cm}^2)$ *A1*

[6 marks]

Method 2

attempt to find the perpendicular height of triangle BAC *(M1)*

EITHER

$\text{height}=\sqrt{15}\times\sin \mathrm{B\hat{A}C}$

attempt to use the Pythagorean identity $\cos^2\alpha+\sin^2\alpha=1$ *(M1)*

$\text{height}=\sqrt{15}\times\sqrt{1-\left(\frac{1}{4}\right)^2}$ *(A1)*

$=\sqrt{15}\times\frac{\sqrt{15}}{4}\left(=\frac{15}{4}\right)$ (may be seen in area formula) *A1*

OR

$\text{adjacent}=\frac{\sqrt{15}}{4}$ *(A1)*

attempt to use Pythagoras' theorem in a right-angled triangle. *(M1)*

height$=\sqrt{15-\frac{15}{16}}\left(=\frac{15}{4}\right)$ (may be seen in area formula) *A1*

THEN

attempt to use 'Area = $\frac{1}{2}$ base $\times$ height' (must include their calculated value for height) ***(M1)***

$= \frac{1}{2} \times 16 \times \frac{15}{4}$

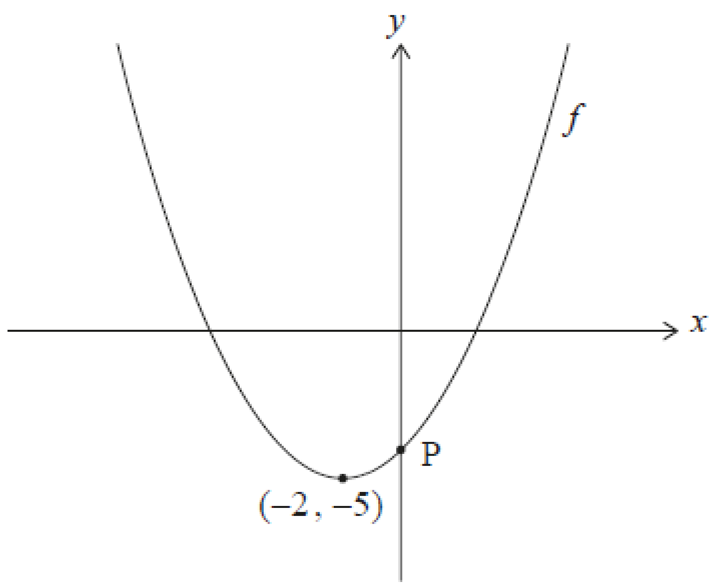
$= 30\ (\mathrm{cm}^2)$ ***A1***

[6 marks]

6. [Maximum mark: 5] 23M.1.SL.TZ2.7

The following diagram shows part of the graph of a quadratic function f.

The vertex of the parabola is $(-2,\ -5)$ and the y-intercept is at point P.



(a) Write down the equation of the axis of symmetry. [1]

Markscheme

$x = -2$ (must be an equation) ***A1***

[1 mark]

The function can be written in the form $f\left(x\right) = \frac{1}{4}(x - h)^2 + k$, where h, $k \in \mathbb{Z}$.

(b) Write down the values of h and k. [2]

Markscheme

$h = -2, k = -5$ ***A1A1***

[2 marks]

(c) Find the y-coordinate of P. [2]

Markscheme

substituting $x = 0$ into $f(x)$ ***(M1)***

$y = \frac{1}{4}(0 + 2)^2 - 5$

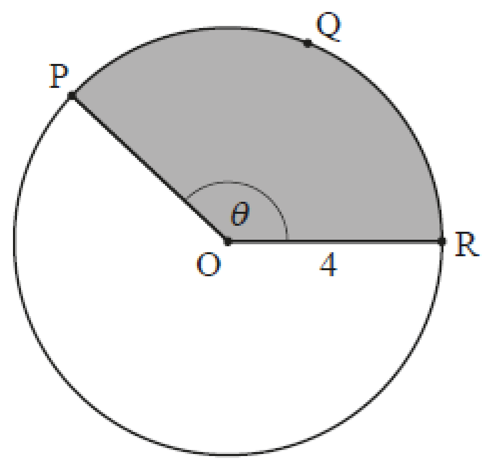
$y = -4$ (accept $\mathrm{P}(0, \ -4)$) ***A1***

[2 marks]

7. [Maximum mark: 6] 23M.1.SL.TZ2.1

The following diagram shows a circle with centre O and radius $4\ \mathrm{cm}$.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\mathrm{P\hat{O}R} = \theta$, where θ is measured in radians.

The length of arc PQR is $10\ \mathrm{cm}$.

(a) Find the perimeter of the shaded sector.

[2]

Markscheme

attempts to find perimeter *(M1)*

$\text{arc} + 2 \times \text{radius}$ OR $10 + 4 + 4$

$= 18\,(\text{cm})$ *A1*

[2 marks]

(b) Find θ.

[2]

Markscheme

$10 = 4\theta$ *(A1)*

$\theta = \frac{10}{4} \left(= \frac{5}{2}, 2.5\right)$ *A1*

[2 marks]

(c) Find the area of the shaded sector. [2]

Markscheme

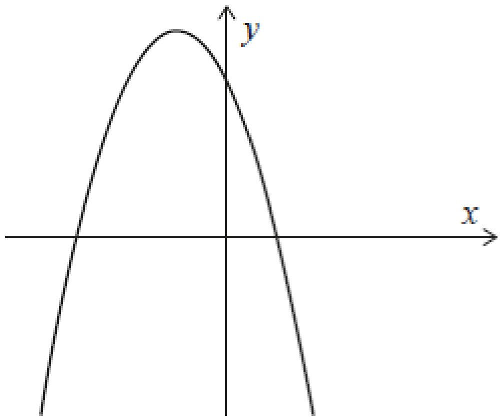
area $= \frac{1}{2}\left(\frac{10}{4}\right)\left(4^2\right) \left(= 1.25 \times 16\right)$ *(A1)*

$= 20 \left(\text{cm}^2\right)$ *A1*

[2 marks]

8. [Maximum mark: 7] 21N.1.SL.TZ0.1

Consider the function $f(x) = -2(x-1)(x+3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



For the graph of f

(a.i) find the x-coordinates of the x-intercepts. [2]

Markscheme

setting $f(x) = 0$ *(M1)*

$x = 1,\ x = -3$ (accept $(1, 0), (-3, 0)$) *A1*

[2 marks]

(a.ii) find the coordinates of the vertex. [3]

Markscheme

METHOD 1

$x = -1$ *A1*

substituting their x-coordinate into f *(M1)*

$y = 8$ *A1*

$(-1, 8)$

METHOD 2

attempt to complete the square *(M1)*

$-2\left((x+1)^2 - 4\right)$ *(M1)*

$x = -1,\ y = 8$ *A1A1*

$(-1, 8)$

[3 marks]

(b) The function f can be written in the form $f(x) = -2(x-h)^2 + k$.

Write down the value of h and the value of k. [2]

Markscheme

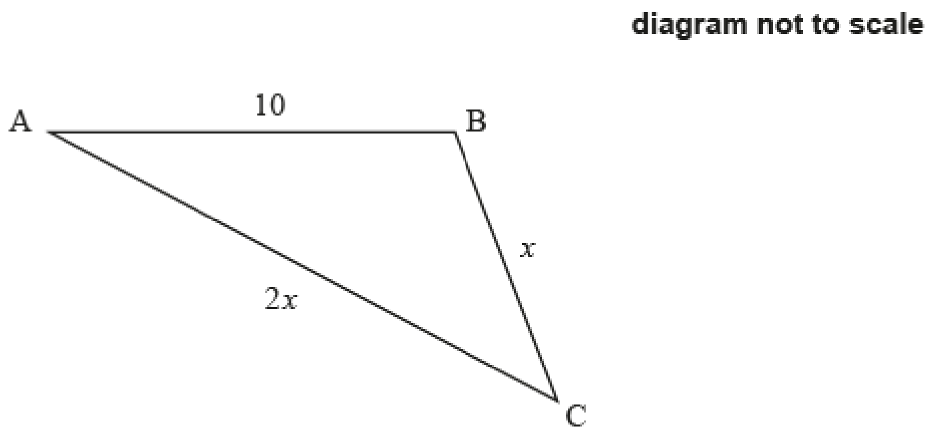
$h = -1$ *A1*

$k = 8$ *A1*

[2 marks]

9. [Maximum mark: 7] 21M.1.SL.TZ2.6

The following diagram shows triangle ABC, with $\mathrm{AB} = 10$, $\mathrm{BC} = x$ and $\mathrm{AC} = 2x$.



Given that $\cos\widehat{\mathrm{C}} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p,\ q \in \mathbb{Z}^+$. [7]

Markscheme

METHOD 1

attempt to use the cosine rule to find the value of x ***(M1)***

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ ***A1***

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}\ \left(= 5\sqrt{2}\right)$ ***A1***

attempt to find $\sin\widehat{C}$ (seen anywhere) ***(M1)***

$\sin^2\widehat{C} + \left(\frac{3}{4}\right)^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side 3 and hypotenuse 4

$\sin\widehat{C} = \frac{\sqrt{7}}{4}$ ***(A1)***

Note: The marks for finding $\sin \widehat{C}$ may be awarded independently of the first three marks for finding x.

correct substitution into the area formula using their value of x (or x^2) and their value of $\sin \widehat{C}$ ***(M1)***

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or
$A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

METHOD 2

attempt to find the height, h, of the triangle in terms of x ***(M1)***

$h^2 + \left(\frac{3}{4}x\right)^2 = x^2$ OR $h^2 + \left(\frac{5}{4}x\right)^2 = 10^2$ OR $h = \frac{\sqrt{7}}{4}x$ ***A1***

equating their expressions for either h^2 or h ***(M1)***

$x^2 - \left(\frac{3}{4}x\right)^2 = 10^2 - \left(\frac{5}{4}x\right)^2$ OR $\sqrt{100 - \frac{25}{16}x^2} = \frac{\sqrt{7}}{4}x$ (or equivalent) ***A1***

$x^2 = 50$ OR $x = \sqrt{50}\ \left(= 5\sqrt{2}\right)$ ***A1***

correct substitution into the area formula using their value of x (or x^2) ***(M1)***

$A = \frac{1}{2} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}\sqrt{50}$ OR
$A = \frac{1}{2}\left(2 \times 5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

[7 marks]

10. [Maximum mark: 6] 21M.1.SL.TZ2.3

(a) Show that the equation $2\cos^2 x + 5\sin x = 4$ may be written in the form $2\sin^2 x - 5\sin x + 2 = 0$. [1]

Markscheme

METHOD 1

correct substitution of $\cos^2 x = 1 - \sin^2 x$ ***A1***

$2(1 - \sin^2 x) + 5\sin x = 4$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

METHOD 2

correct substitution using double-angle identities ***A1***

$(2\cos^2 x - 1) + 5\sin x = 3$

$1 - 2\sin^2 x - 5\sin x = 3$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

[1 mark]

(b) Hence, solve the equation $2\cos^2 x + 5\sin x = 4,\ 0 \le x \le 2\pi$. [5]

Markscheme

EITHER

attempting to factorise ***M1***

$(2\sin x - 1)(\sin x - 2)$ ***A1***

OR

attempting to use the quadratic formula ***M1***

$\sin x = \frac{5 \pm \sqrt{5^2 - 4 \times 2 \times 2}}{4} \left(= \frac{5 \pm 3}{4}\right)$ ***A1***

THEN

$\sin x = \frac{1}{2}$ ***(A1)***

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ ***A1A1***

[5 marks]

11. [Maximum mark: 6] 21M.1.SL.TZ2.1

The following diagram shows a circle with centre O and radius r.

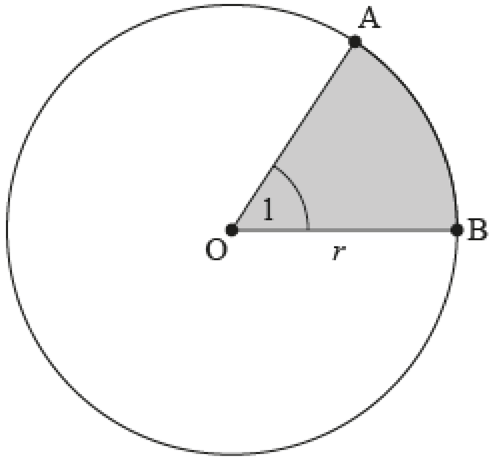


diagram not to scale

Points A and B lie on the circumference of the circle, and $\mathrm{A}\hat{\mathrm{O}}\mathrm{B} = 1$ radian.

The perimeter of the shaded region is 12.

(a) Find the value of r. [3]

Markscheme

minor arc AB has length r ***(A1)***

recognition that perimeter of shaded sector is $3r$ ***(A1)***

$3r = 12$

$r = 4$ ***A1***

[3 marks]

(b) Hence, find the exact area of the **non-shaded** region. [3]

Markscheme

EITHER

$\theta = 2\pi - \mathrm{A\widehat{O}B} \, (= 2\pi - 1)$ ***(M1)***

Area of non-shaded region $= \frac{1}{2}(2\pi - 1)\left(4^2\right)$ ***(A1)***

OR

area of circle — area of shaded sector ***(M1)***

$16\pi - \left(\frac{1}{2} \times 1 \times 4^2\right)$ ***(A1)***

THEN

area $= 16\pi - 8 \ (= 8(2\pi - 1))$ ***A1***

[3 marks]