Kinematics P1 [66 marks]

1. [Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\text{m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0$, P is at the origin O.

(a) Find the value of t when P reaches its maximum velocity. [2]

Markscheme

valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

[2 marks]

(b) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [5]

Markscheme

attempt to integrate v ***(M1)***

$\int v \, \text{d}\, t = \int \left(4 + 4t - 3t^2\right) \, \text{d}\, t = 4t + 2t^2 - t^3 (+c)$

A1A1

Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

distance $= 4\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

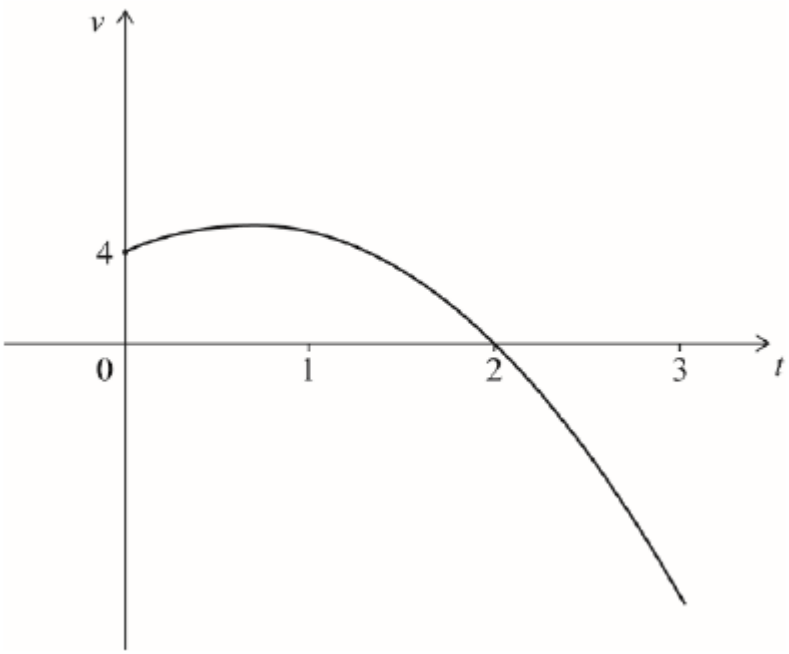
$= \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

$= \frac{88}{27}$ (m) ***AG***

[5 marks]

(c) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

Markscheme



valid approach to solve $4 + 4t - 3t^2 = 0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\frac{-4\pm\sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t=2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0, 4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t=\frac{2}{3}$ and $v>4$ ***A1***

[4 marks]

(d) Find the total distance travelled by P. [5]

Markscheme

recognising to integrate between 0 and 2, or 2 and 3 OR

$\int_0^3 \left|4+4t-3t^2\right| \, \mathrm{d}\, t$ ***(M1)***

$\int_0^2 \left(4+4t-3t^2\right) \, \mathrm{d}\, t$

$=8$ ***A1***

$\int_2^3 \left(4+4t-3t^2\right) \, \mathrm{d}\, t$

$= -5$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \, \mathrm{d}\, t - \int_2^3 v \, \mathrm{d}\, t$ OR $\int_0^2 v \, \mathrm{d}\, t + \left| \int_2^3 v \, \mathrm{d}\, t \right|$

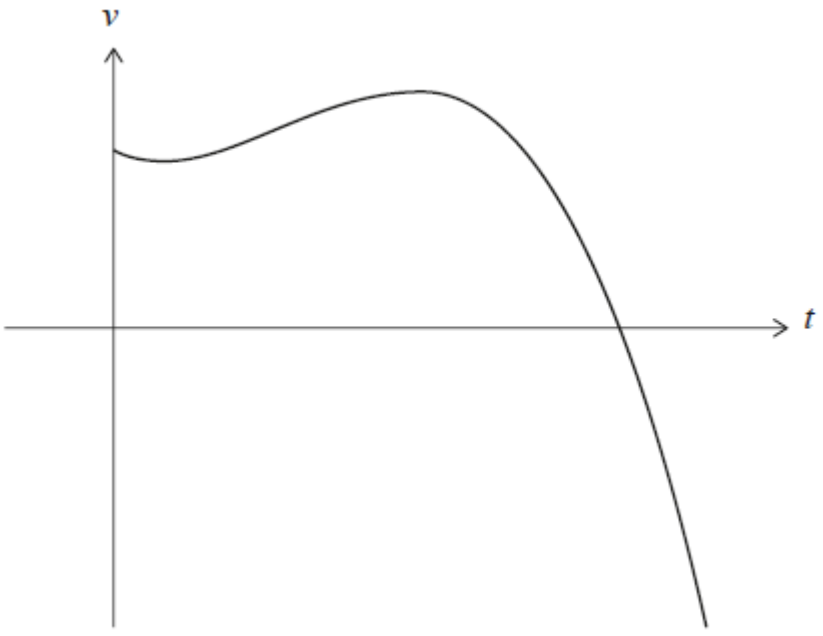
total distance travelled $= 13$ (m) ***A1***

[5 marks]

2. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v \, \mathrm{m\,s^{-1}}$, at time t seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \leq t \leq 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$. [5]

Markscheme

attempt to integrate v (integration of at least one term) ***(M1)***

$(s(t) =) -\frac{1}{4}t^4 + \frac{7}{6}t^3 - t^2 + 6t\ (+C)$ ***A2***

Note: Award ***A1*** for at least two correct terms.

substitution of $t = 1$ into their integrated expression ***(M1)***

displacement $= 5\frac{11}{12}\left(= \frac{71}{12}\right)(\text{m})$ ***A1***

[5 marks]

(b) Find an expression for the acceleration of the object. [2]

Markscheme

attempt to differentiate v (differentiation of at least one term) ***(M1)***

$a(t) = -3t^2 + 7t - 2$ ***A1***

[2 marks]

(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

Markscheme

setting their $v'(t) = 0$ ***(M1)***

$-3t^2 + 7t - 2 = 0$

valid attempt to solve quadratic ***(M1)***

$(3t - 1)(t - 2) = 0$ OR $\frac{-7 \pm \sqrt{49 - 4(-3)(-2)}}{-6}$

$t = \frac{1}{3},\ 2$ ($t = \frac{1}{3}$ may be omitted) ***(A1)***

substitute their largest positive t-value into $v(t)$ ***(M1)***

greatest speed is $8\ (\text{ms}^{-1})$ ***A1***

[5 marks]

(d) Find the greatest speed reached by the object for $0 \leq t \leq 4$. [2]

Markscheme

attempt to check other boundary value at $t = 4$ ***(M1)***

$v(4) = -64 + 56 - 8 + 6\ (= -10)$

greatest speed is $10\ \text{ms}^{-1}$ ***A1***

[2 marks]

(e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

Markscheme

identifying correct intervals where speed increases (may be seen in integral) ***(A1)(A1)***

$t = \frac{1}{3}$ to $t = 2$ and $t = k$ to $t = 4$

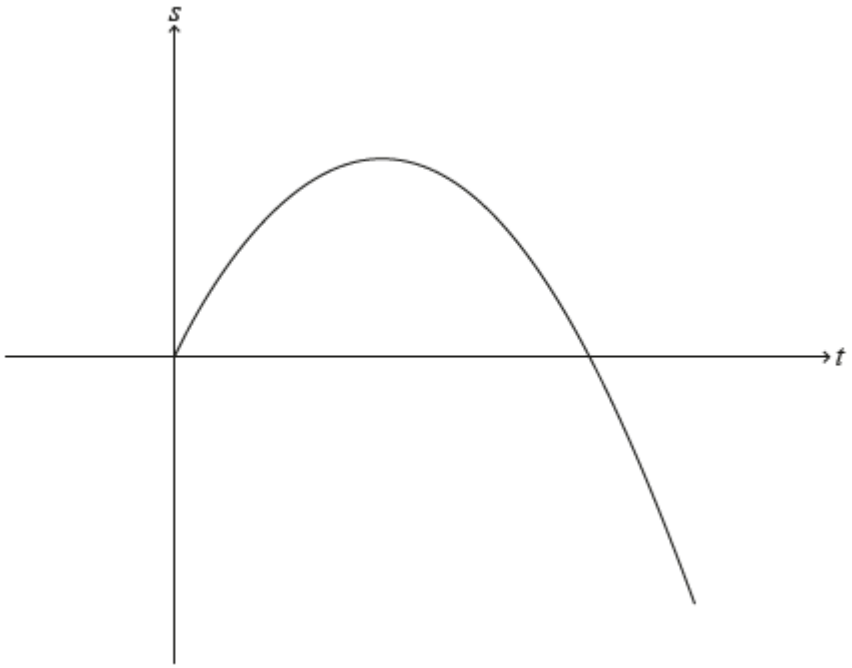
$\int_{\frac{1}{3}}^{2} v\left(t\right)\mathrm{d}t + \int_{k}^{4} |v(t)|\mathrm{d}t$ OR $\int_{\frac{1}{3}}^{2} v\mathrm{d}t + \left|\int_{\mathrm{k}}^{4} \mathrm{v}\mathrm{d}\,t\right|$ OR $\int_{\frac{1}{3}}^{2} v\left(t\right)\mathrm{d}t - \int_{k}^{4} v\left(t\right)\mathrm{d}t$ ***A1***

Note: Condone missing $\mathrm{d}t$.

[3 marks]

3. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s\left(t\right) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.
Particle A changes direction when $t = q$.

The total distance travelled by particle A is given by d.

(a) Find the value of p. [2]

Markscheme

setting $s(t) = 0$ ***(M1)***

$8t - t^2 = 0$

$t(8 - t) = 0$

$p = 8$ (accept $t = 8,\ (8, 0)$) ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $p = 0, 8$).

[2 marks]

(b) Find the value of q. [2]

Markscheme

recognition that when particle changes direction $v = 0$ OR local maximum on graph of s OR vertex of parabola *(M1)*

$q = 4$ (accept $t = 4$) *A1*

[2 marks]

(c) Find the displacement of particle A from the origin when $t = q$. [2]

Markscheme

substituting their value of q into $s(t)$ OR integrating $v(t)$ from $t = 0$ to $t = 4$ *(M1)*

$\text{displacement} = 16\,(\text{m})$ *A1*

[2 marks]

(d) Find the distance of particle A from the origin when $t = 10$. [2]

Markscheme

$s(10) = -20$ OR $\text{distance} = |\text{s}(t)|$ OR integrating $v(t)$ from $t = 0$ to $t = 10$ ***(M1)***

$\text{distance} = 20\,(\text{m})$ ***A1***

[2 marks]

(e) Find the value of d. [2]

Markscheme

16 forward $+ 36$ backward OR $16 + 16 + 20$ OR $\int_0^{10} |v(t)| \text{ d } t$ ***(M1)***

$d = 52\,(\text{m})$ ***A1***

[2 marks]

(f) A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d.

Find the value of k. [4]

Markscheme

METHOD 1

graphical method with triangles on $v(t)$ graph ***M1***

$49 + \left(\frac{x(2x)}{2}\right)$ *(A1)*

$49 + x^2 = 52,\ x = \sqrt{3}$ *(A1)*

$k = 7 + \sqrt{3}$ *A1*

METHOD 2

recognition that distance $= \int |v(t)| \ \mathrm{d}\ t$ *M1*

$\int_0^7 (14 - 2t) \ \mathrm{d}\ t + \int_7^k (2t - 14) \ \mathrm{d}\ t$

$\left[14t - t^2\right]_0^7 + \left[t^2 - 14t\right]_7^k$ *(A1)*

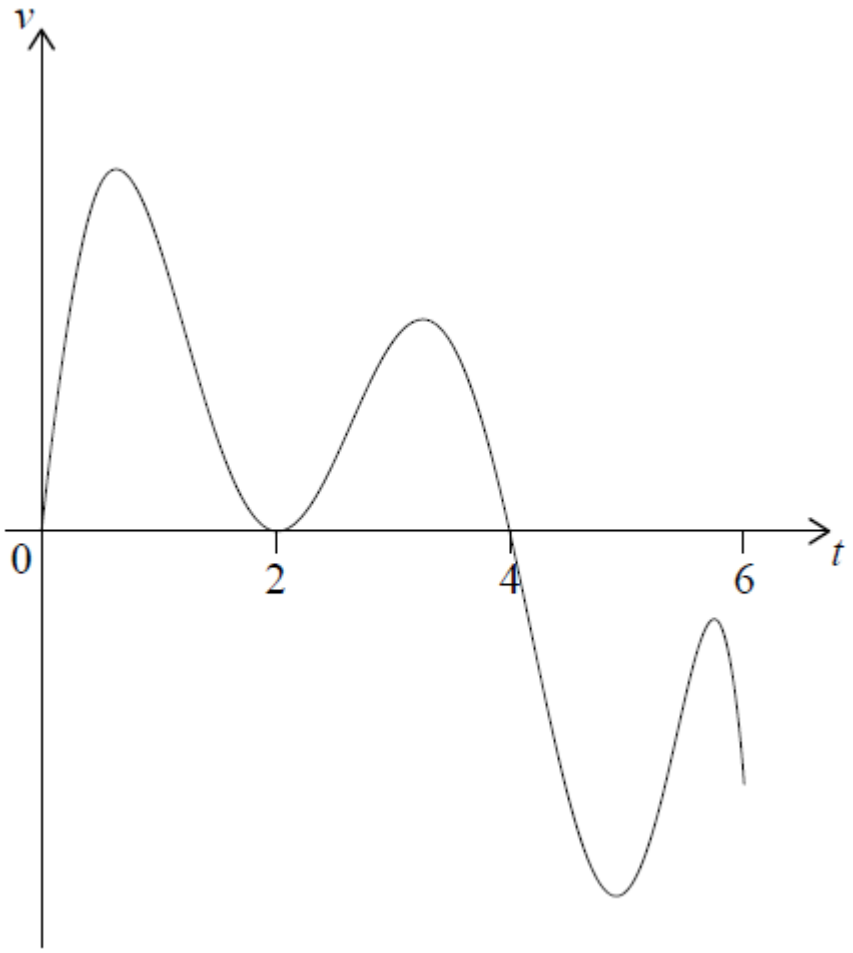
$14(7) - 7^2 + \left(\left(k^2 - 14k\right) - \left(7^2 - 14(7)\right)\right) = 52$ *(A1)*

$k = 7 + \sqrt{3}$ *A1*

[4 marks]

4. [Maximum mark: 7]

A particle P starts from point O and moves along a straight line. The graph of its velocity, v ms^{-1} after t seconds, for $0 \leq t \leq 6$, is shown in the following diagram.



The graph of v has t-intercepts when $t = 0$, 2 and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v \, dt = 9$.

(a) Find the value of $s(4) - s(2)$. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing relationship between v and s ***(M1)***

eg $\int v = s, s' = v$

$s(4) - s(2) = 9$ ***A1 N2***

[2 marks]

(b) Find the total distance travelled in the first 5 seconds. [5]

Markscheme

correctly interpreting distance travelled in first 2 seconds (seen anywhere, including part (a) or the area of 15 indicated on diagram) ***(A1)***

eg $\int_0^2 v = 15, s(2) = 15$

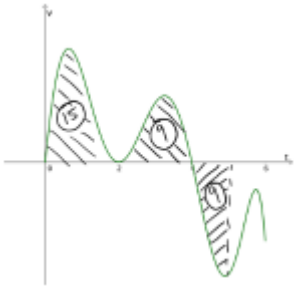
valid approach to find total distance travelled ***(M1)***

eg sum of 3 areas, $\int_0^4 v + \int_4^5 v$, shaded areas in diagram between 0 and 5

Note: Award ***M0*** if only $\int_0^5 |v|$ is seen.

correct working towards finding distance travelled between 2 and 5 (seen anywhere including within total area expression or on diagram) ***(A1)***

eg $\int_2^4 v - \int_4^5 v$, $\int_2^4 v = \int_4^5 |v|$, $\int_4^5 v \, \mathrm{d}t = -9$,
$s(4) - s(2) - [s(5) - s(4)]$,

equal areas 

correct working using $s(5) = s(2)$ ***(A1)***

eg $15 + 9 - (-9)$, $15 + 2[s(4) - s(2)]$, $15 + 2(9)$,
$2 \times s(4) - s(2)$, $48 - 15$

total distance travelled = 33 (m) ***A1 N2***

[5 marks]

5. [Maximum mark: 5]

A particle moves in a straight line such that at time t seconds $(t \geqslant 0)$, its velocity v, in ms^{-1}, is given by $v = 10te^{-2t}$. Find the exact distance travelled by the particle in the first half-second. [5]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$$s = \int_0^{\frac{1}{2}} 10te^{-2t}\mathrm{d}t$$

attempt at integration by parts ***M1***

$$= \left[-5te^{-2t}\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} -5e^{-2t}\mathrm{d}t \quad \textbf{A1}$$

$$= \left[-5te^{-2t} - \tfrac{5}{2}e^{-2t}\right]_0^{\frac{1}{2}} \quad \textbf{(A1)}$$

Note: Condone absence of limits (or incorrect limits) and missing factor of 10 up to this point.

$$s = \int_0^{\frac{1}{2}} 10te^{-2t}\mathrm{d}t \quad \textbf{(M1)}$$

$= -5\mathrm{e}^{-1} + \frac{5}{2}\ \left(= \frac{-5}{\mathrm{e}} + \frac{5}{2}\right)\ \left(= \frac{5\mathrm{e}-10}{2\mathrm{e}}\right)$ ***A1***

[5 marks]

6. [Maximum mark: 7]

A particle moves along a straight line. Its displacement, s metres, at time t seconds is given by $s = t + \cos 2t,\ t \geqslant 0$. The first two times when the particle is at rest are denoted by t_1 and t_2, where $t_1 < t_2$.

(a) Find t_1 and t_2. [5]

Markscheme

*This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$s = t + \cos 2t$

$\frac{\mathrm{d}s}{\mathrm{d}t} = 1 - 2\sin 2t$ ***M1A1***

$= 0$ ***M1***

$\Rightarrow \sin 2t = \frac{1}{2}$

$t_1 = \frac{\pi}{12}(s),\ t_2 = \frac{5\pi}{12}(s)$ ***A1A1***

Note: Award ***A0A0*** if answers are given in degrees.

[5 marks]

(b) Find the displacement of the particle when $t = t_1$ [2]

Markscheme

$s = \frac{\pi}{12} + \cos\frac{\pi}{6}$ $\left(s = \frac{\pi}{12} + \frac{\sqrt{3}}{2}(m)\right)$ **A1A1**

[2 marks]