

Kinematics P1 [66 marks]

1. [Maximum mark: 16]

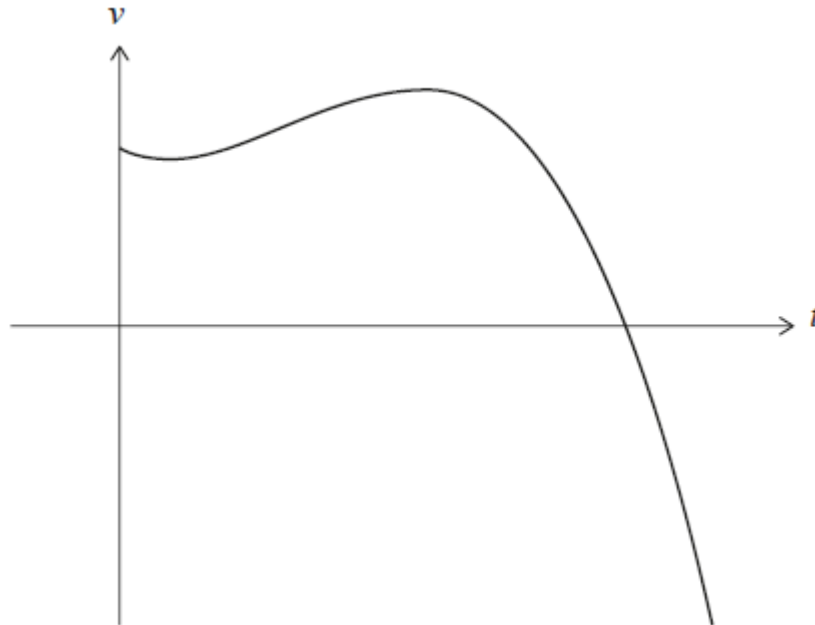
A particle P moves along the x -axis. The velocity of P is $v \text{ m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0$, P is at the origin O .

- (a) Find the value of t when P reaches its maximum velocity. [2]
- (b) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [5]
- (c) Sketch a graph of v against t , clearly showing any points of intersection with the axes. [4]
- (d) Find the total distance travelled by P . [5]

2. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \leq t \leq 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.

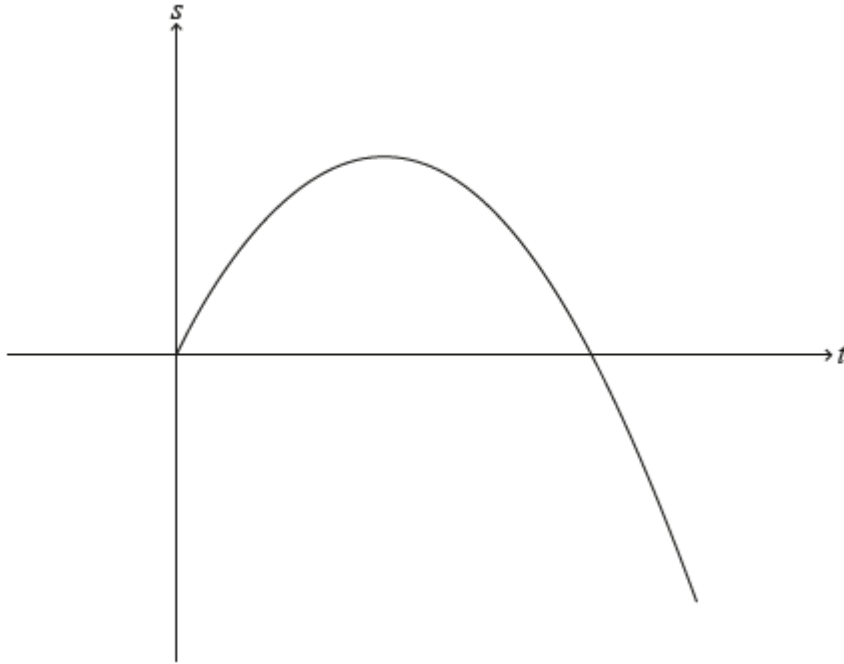


At $t = 0$, the object is at the origin.

- (a) Find the displacement of the object from the origin at $t = 1$. [5]
- (b) Find an expression for the acceleration of the object. [2]
- (c) Hence, find the greatest speed reached by the object before it comes to rest. [5]
- (d) Find the greatest speed reached by the object for $0 \leq t \leq 4$. [2]
- (e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

3. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.
 Particle A changes direction when $t = q$.

The total distance travelled by particle A is given by d .

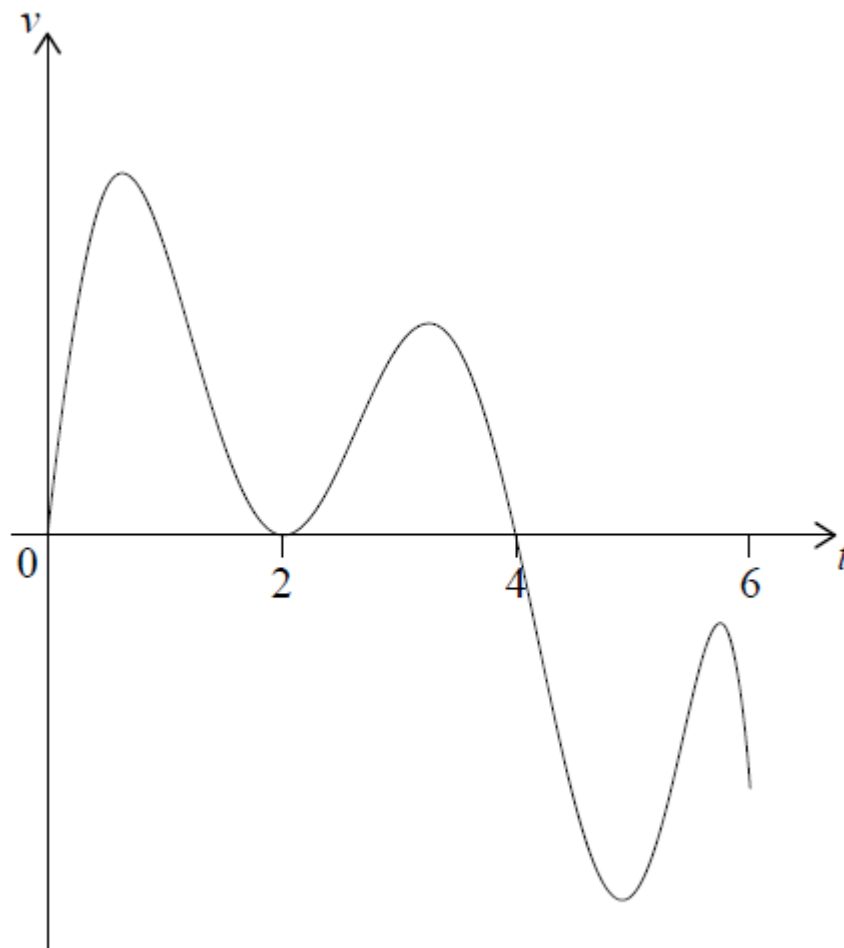
- (a) Find the value of p . [2]
- (b) Find the value of q . [2]
- (c) Find the displacement of particle A from the origin when $t = q$. [2]
- (d) Find the distance of particle A from the origin when $t = 10$. [2]
- (e) Find the value of d . [2]
- (f) A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d .

Find the value of k .

4. [Maximum mark: 7]

A particle P starts from point O and moves along a straight line. The graph of its velocity, $v \text{ ms}^{-1}$ after t seconds, for $0 \leq t \leq 6$, is shown in the following diagram.



The graph of v has t -intercepts when $t = 0, 2$ and 4 .

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v \, dt = 9$.

(a) Find the value of $s(4) - s(2)$. [2]

(b) Find the total distance travelled in the first 5 seconds. [5]

5. [Maximum mark: 5]

A particle moves in a straight line such that at time t seconds ($t \geq 0$), its velocity v , in ms^{-1} , is given by $v = 10te^{-2t}$. Find the exact distance travelled by the particle in the first half-second. [5]

6. [Maximum mark: 7]

A particle moves along a straight line. Its displacement, s metres, at time t seconds is given by $s = t + \cos 2t$, $t \geq 0$. The first two times when the particle is at rest are denoted by t_1 and t_2 , where $t_1 < t_2$.

(a) Find t_1 and t_2 . [5]

(b) Find the displacement of the particle when $t = t_1$ [2]

Kinematics P2 [96 marks]

1. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O , at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \leq t \leq 5$.

(a) Find the maximum distance of the particle from O . [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

2. [Maximum mark: 5]

A particle moves in a straight line such that its velocity, $v \text{ ms}^{-1}$, at time t seconds is given by

$$v = 4t^2 - 6t + 9 - 2 \sin(4t), \quad 0 \leq t \leq 1.$$

The particle's acceleration is zero at $t = T$.

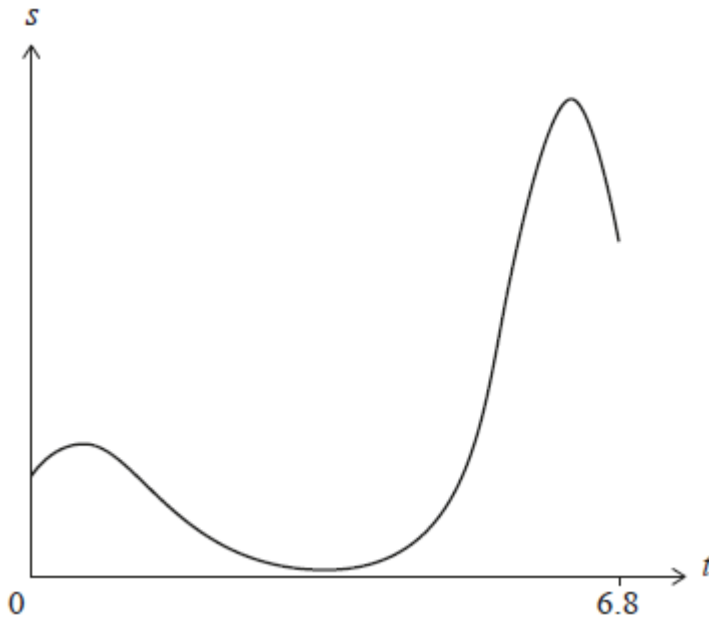
(a) Find the value of T . [2]

(b) Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$.

Show that $s_2 > s_1$. [3]

3. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s(t) = 3(t + 2)^{\cos t}$, for $0 \leq t \leq 6.8$, as shown in the following graph.



The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

- (a) Find the particle's initial displacement from the point P. [2]
- (b) Find the particle's velocity when $t = 2$. [2]
- (c) Determine the intervals of time when the particle is moving away from the point P. [5]
- (d) Find the value of b and the value of c . [4]
- (e) Find the total distance travelled by the particle for $b \leq t \leq c$. [3]

4. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, after t seconds is given by $v(t) = e^{\sin t} + 4 \sin t$ for $0 \leq t \leq 6$.

- (a) Find the value of t when the particle is at rest. [2]
- (b) Find the acceleration of the particle when it changes direction. [3]
- (c) Find the total distance travelled by the particle. [2]

5. [Maximum mark: 7]

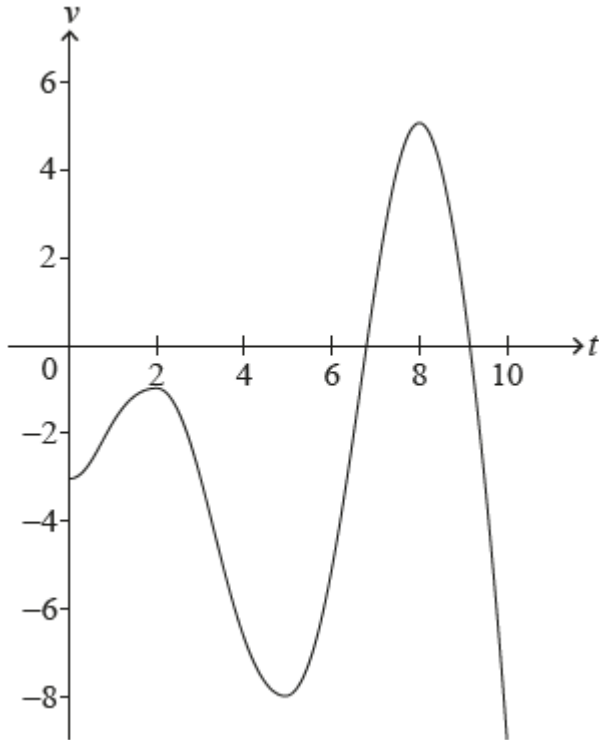
A particle moves in a straight line such that its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by $v = \frac{(t^2+1) \cos t}{4}$, $0 \leq t \leq 3$.

- (a) Determine when the particle changes its direction of motion. [2]
- (b) Find the times when the particle's acceleration is -1.9 m s^{-2} . [3]
- (c) Find the particle's acceleration when its speed is at its greatest. [2]

6. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v \text{ m s}^{-1}$, of the particle at time t seconds is given by $v(t) = t \sin t - 3$, for $0 \leq t \leq 10$.

The following diagram shows the graph of v .



- (a) Find the smallest value of t for which the particle is at rest. [2]
- (b) Find the total distance travelled by the particle. [2]
- (c) Find the acceleration of the particle when $t = 7$. [2]

7. [Maximum mark: 20]

A particle P moves in a straight line such that after time t seconds, its velocity, v in ms^{-1} , is given by $v = e^{-3t} \sin 6t$, where $0 < t < \frac{\pi}{2}$.

At time t , P has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

At successive times when the acceleration of P is 0ms^{-2} , the velocities of P form a geometric sequence. The acceleration of P is zero at times t_1 , t_2 , t_3 where $t_1 < t_2 < t_3$ and the respective velocities are v_1 , v_2 , v_3 .

- (a) Find the times when P comes to instantaneous rest. [2]

- (b) Find an expression for s in terms of t . [7]
- (c) Find the maximum displacement of P , in metres, from its initial position. [2]
- (d) Find the total distance travelled by P in the first 1.5 seconds of its motion. [2]
- (e) Show that, at these times, $\tan 6t = 2$. [2]
- (f) Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -e^{-\frac{\pi}{2}}$. [5]

8. [Maximum mark: 14]

A rocket is travelling in a straight line, with an initial velocity of 140 m s^{-1} . It accelerates to a new velocity of 500 m s^{-1} in two stages.

During the first stage its acceleration, $a \text{ m s}^{-2}$, after t seconds is given by $a(t) = 240 \sin(2t)$, where $0 \leq t \leq k$.

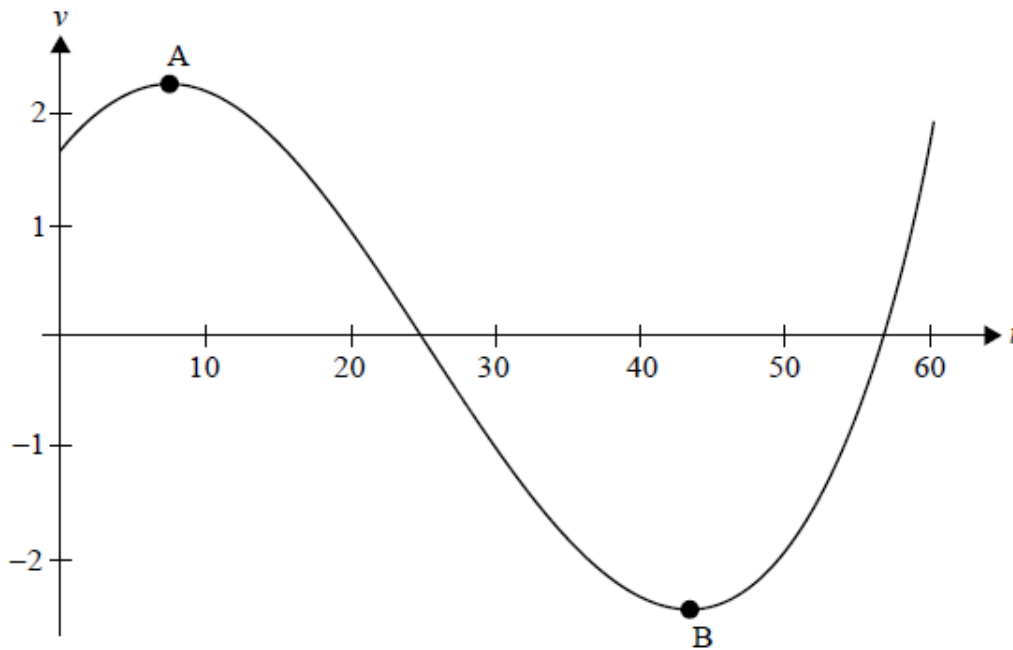
The first stage continues for k seconds until the velocity of the rocket reaches 375 m s^{-1} .

- (a) Find an expression for the velocity, $v \text{ m s}^{-1}$, of the rocket during the first stage. [4]
- (b) Find the distance that the rocket travels during the first stage. [4]
- (c) During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage.
- Find the total time taken for the two stages. [6]

9. [Maximum mark: 14]

A body moves in a straight line such that its velocity, $v \text{ ms}^{-1}$, after t seconds is given by $v = 2 \sin \left(\frac{t}{10} + \frac{\pi}{5} \right) \csc \left(\frac{t}{30} + \frac{\pi}{4} \right)$ for $0 \leq t \leq 60$.

The following diagram shows the graph of v against t . Point **A** is a local maximum and point **B** is a local minimum.



The body first comes to rest at time $t = t_1$. Find

- (a) Determine the coordinates of point **A** and the coordinates of point **B**. [4]
- (b) Hence, write down the maximum speed of the body. [1]
- (c) the value of t_1 . [2]
- (d) the distance travelled between $t = 0$ and $t = t_1$. [2]
- (e) the acceleration when $t = t_1$. [2]
- (f) Find the distance travelled in the first 30 seconds. [3]

