Kinematics P2 [96 marks]

1. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \leq t \leq 5$.

(a) Find the maximum distance of the particle from O. [3]

Markscheme

use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is 16.5 cm (to the left of O) ***A1***

[3 marks]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

Markscheme

attempt to find time when particle changes direction *eg* considering the first maximum on the graph of s or the first t – intercept on the graph of s'. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of s' for **their** value of t, $s''(1.51986...)$ ***(M1)***

$= -8.92$ (cm/s^2) ***A1***

[4 marks]

2. [Maximum mark: 5]

A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by

$$v = 4t^2 - 6t + 9 - 2\sin(4t),\ 0 \leq t \leq 1.$$

The particle's acceleration is zero at $t = T$.

(a) Find the value of T. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting

compared to formal exam papers.

attempts either graphical or symbolic means to find the value of t when $\frac{dv}{dt} = 0$ **(M1)**

$T = 0.465\,(\text{s})$ **A1**

[2 marks]

(b) Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$.

Show that $s_2 > s_1$. [3]

Markscheme

attempts to find the value of either $s_1 = \int_0^{0.46494\ldots} v\,dt$ or $s_2 = \int_{0.46494\ldots}^{1} v\,dt$ **(M1)**

$s_1 = 3.02758\ldots$ and $s_2 = 3.47892\ldots$ **A1A1**

Note: Award as above for obtaining, for example, $s_2 - s_1 = 0.45133\ldots$ or $\frac{s_2}{s_1} = 1.14907\ldots$

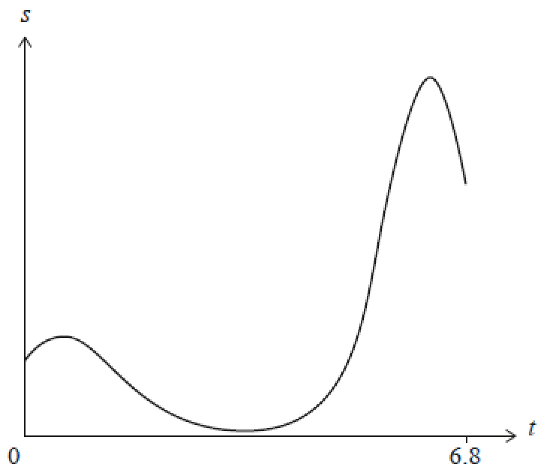
Note: Award a maximum of **M1A1A0FT** for use of an incorrect value of T from part (a).

so $s_2 > s_1$ **AG**

[3 marks]

3. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s\left(t\right) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.



The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(a) Find the particle's initial displacement from the point P. [2]

Markscheme

initial displacement is $s(0)$ *(M1)*

$6\,(\mathrm{m})$ *A1*

[2 marks]

(b) Find the particle's velocity when $t = 2$. [2]

Markscheme

velocity is $s\prime$ *(M1)*

-2.29920

$-2.30\ (\mathrm{m/s})$ *A1*

[2 marks]

(c) Determine the intervals of time when the particle is moving away from the point P. [5]

Markscheme

attempting to find t when the particle changes direction *(M1)*

$t = 0.433007\ldots$ AND $3.25575\ldots$ AND $6.33965\ldots$ (may be seen on a graph) *(A1)* particle travels away from P when $v > 0$ OR when $s\prime > 0$ *(M1)*

$0 \leq t < 0.433007\ldots,\ 3.25575\ldots < t < 6.33965$

$0 \leq t < 0.433,\ 3.26 < t < 6.34$ ***A1A1***

[5 marks]

(d) Find the value of b and the value of c. [4]

Markscheme

recognizing that acceleration is $a(t) = v\prime(t)$ OR $a(t) = s\prime\prime(t)$ ***(M1)***

attempting to find max/min on graph of velocity OR finding zeros on graph of acceleration ***(M1)***

$b = 1.23140\ldots,\ c = 5.68959\ldots$

$b = 1.23,\ c = 5.69$ ***A1A1***

[4 marks]

(e) Find the total distance travelled by the particle for $b \leq t \leq c$. [3]

Markscheme

METHOD 1 (using integral of velocity)

correct integral (accept absence of $\mathrm{d}\ t$) ***(A1)***

$\int_{1.23140\ldots}^{5.68959\ldots} |v(t)|\ \mathrm{d}\ t$ OR $\int_b^c |s\prime(t)|\ \mathrm{d}\ t$ OR $-\int_{1.23140\ldots}^{3.25575\ldots} v(t)\ \mathrm{d}\ t + \int_{3.25575\ldots}^{5.68959\ldots} v(t)\ \mathrm{d}\ t$ OR $3.8560 + 15.696$

$19.5525\ldots$

total distance $= 19.6\ (\mathrm{m})$ ***A2***

METHOD 2 (using differences in displacement)

finding displacement at b, c **and** local min on displacement graph ***(A1)***

$(b,\ 4.43306),\ (c,\ 16.2734),\ (3.25575,\ 0.577001)$ OR $4.43306,\ 0.577001,\ 16.2734$

correct approach ***(A1)***

$(4.43306 - 0.577001) + (16.2734 - 0.577001)$ OR towards $\mathrm{P}\ 3.85606 +$ away from $\mathrm{P}\ 15.696$

$19.5525\ldots$

total distance $= 19.6\ (\text{m})$ ***A1***

[3 marks]

4. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\text{m}\,\text{s}^{-1}$, after t seconds is given by $v(t) = \text{e}^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

(a) Find the value of t when the particle is at rest. [2]

Markscheme

recognizing at rest $v = 0$ ***(M1)***

$t = 3.34692\ldots$

$t = 3.35$ (seconds) ***A1***

Note: Award ***(M1)A0*** for additional solutions to $v = 0$ eg $t = -0.205$ or $t = 6.08$.

[2 marks]

(b) Find the acceleration of the particle when it changes direction. [3]

Markscheme

recognizing particle changes direction when $v = 0$ OR when $t = 3.34692\ldots$ ***(M1)***

$a = -4.71439\ldots$

$a = -4.71\ (\text{ms}^{-2})$ ***A2***

[3 marks]

(c) Find the total distance travelled by the particle. [2]

Markscheme

distance travelled $= \int_0^6 |v| \, \mathrm{d}\, t$ OR

$\int_0^{3.34...} \left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right) \mathrm{d}\, t - \int_{3.34...}^{6} \left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right) \mathrm{d}\, t \ (= 14.3104... + 6.44300...)$ ***(A1)***

$= 20.7534...$

$= 20.8$ (metres) ***A1***

[2 marks]

5. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v \, \mathrm{m\,s^{-1}}$, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4}, \ 0 \le t \le 3.$

(a) Determine when the particle changes its direction of motion. [2]

Markscheme

recognises the need to find the value of t when $v = 0$ ***(M1)***

$t = 1.57079... \left(= \frac{\pi}{2}\right)$

$t = 1.57 \left(= \frac{\pi}{2}\right)$ (s) ***A1***

[2 marks]

(b) Find the times when the particle's acceleration is $-1.9 \, \mathrm{m\,s^{-2}}$. [3]

Markscheme

recognises that $a(t) = v\prime(t)$ ***(M1)***

$t_1 = 2.26277..., \ t_2 = 2.95736...$

$t_1 = 2.26, \ t_2 = 2.96$ (s) ***A1A1***

Note: Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \le t \le 3$.

[3 marks]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

Markscheme

speed is greatest at $t = 3$ ***(A1)***

$a = -1.83778\ldots$

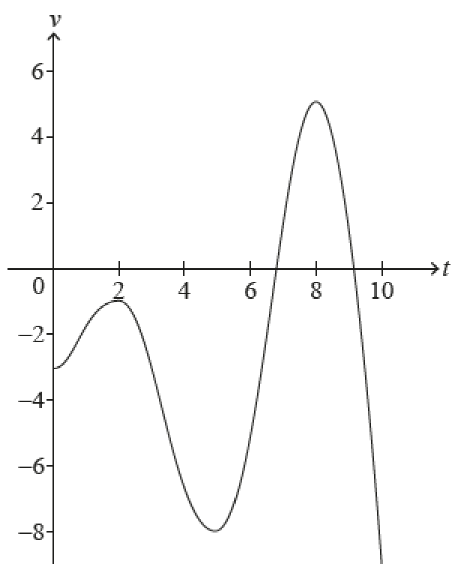
$a = -1.84\ (\mathrm{ms}^{-2})$ ***A1***

[2 marks]

6. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\,\mathrm{ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of v.



(a) Find the smallest value of t for which the particle is at rest. [2]

Markscheme

recognising $v = 0$ ***(M1)***

$t = 6.74416\ldots$

$= 6.74$ (sec) ***A1***

Note: Do not award ***A1*** if additional values are given.

[2 marks]

(b) Find the total distance travelled by the particle. [2]

Markscheme

$\int_0^{10}|v(t)|\ \mathrm{d}\,t$ OR $-\int_0^{6.74416\ldots} v(t)\ \mathrm{d}\,t+\int_{6.74416\ldots}^{9.08837\ldots} v(t)\ \mathrm{d}\,t-\int_{9.08837\ldots}^{10} v(t)\ \mathrm{d}\,t$ ***(A1)***

$=37.0968\ldots$

$=37.1\ (\mathrm{m})$ ***A1***

[2 marks]

(c) Find the acceleration of the particle when $t=7$. [2]

Markscheme

recognizing acceleration at $t=7$ is given by $v\prime(7)$ ***(M1)***

acceleration $=5.93430\ldots$

$=5.93\ \left(\mathrm{ms}^{-2}\right)$ ***A1***

[2 marks]

7. [Maximum mark: 20]

A particle P moves in a straight line such that after time t seconds, its velocity, v in $\mathrm{m\,s}^{-1}$, is given by $v=\mathrm{e}^{-3t}\sin 6t$, where $0<t<\frac{\pi}{2}$.

At time t, P has displacement $s(t)$; at time $t=0$, $s(0)=0$.

At successive times when the acceleration of P is $0\,\mathrm{m\,s}^{-2}$, the velocities of P form a geometric sequence. The acceleration of P is zero at times $t_1,\ t_2,\ t_3$ where $t_1<t_2<t_3$ and the respective velocities are $v_1,\ v_2,\ v_3$.

(a) Find the times when P comes to instantaneous rest. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\pi}{6}(=0.524)$ ***A1***

$\frac{\pi}{3}(=1.05)$ ***A1***

[2 marks]

(b) Find an expression for s in terms of t. [7]

Markscheme

attempt to use integration by parts ***M1***

$s=\int \mathrm{e}^{-3t}\sin 6t\,\mathrm{d}t$

EITHER

$=-\frac{\mathrm{e}^{-3t}\sin 6t}{3}-\int -2\mathrm{e}^{-3t}\cos 6t\,\mathrm{d}t$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\sin 6t}{3}-\left(\frac{2\mathrm{e}^{-3t}\cos 6t}{3}-\int -4\mathrm{e}^{-3t}\sin 6t\,\mathrm{d}t\right)$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\sin 6t}{3}-\left(\frac{2\mathrm{e}^{-3t}\cos 6t}{3}+4s\right)$

$5s=\frac{-3\mathrm{e}^{-3t}\sin 6t-6\mathrm{e}^{-3t}\cos 6t}{9}$ ***M1***

OR

$=-\frac{\mathrm{e}^{-3t}\cos 6t}{6}-\int \frac{1}{2}\mathrm{e}^{-3t}\cos 6t\,\mathrm{d}t$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\cos 6t}{6}-\left(\frac{\mathrm{e}^{-3t}\sin 6t}{12}+\int \frac{1}{4}\mathrm{e}^{-3t}\sin 6t\,\mathrm{d}t\right)$ ***A1***

$=-\frac{\mathrm{e}^{-3t}\cos 6t}{6}-\left(\frac{\mathrm{e}^{-3t}\sin 6t}{12}+\frac{1}{4}s\right)$

$\frac{5}{4}s=\frac{-2\mathrm{e}^{-3t}\cos 6t-\mathrm{e}^{-3t}\sin 6t}{12}$ ***M1***

THEN

$s = -\frac{e^{-3t}(\sin 6t + 2\cos 6t)}{15} (+c)$ ***A1***

at $t = 0,\ s = 0 \Rightarrow 0 = -\frac{2}{15} + c$ ***M1***

$c = \frac{2}{15}$ ***A1***

$s = \frac{2}{15} - \frac{e^{-3t}(\sin 6t + 2\cos 6t)}{15}$

[7 marks]

(c) Find the maximum displacement of P, in metres, from its initial position. [2]

Markscheme

EITHER

substituting $t = \frac{\pi}{6}$ into their equation for s ***(M1)***

$\left(s = \frac{2}{15} - \frac{e^{-\frac{\pi}{2}}(\sin \pi + 2\cos \pi)}{15}\right)$

OR

using GDC to find maximum value ***(M1)***

OR

evaluating $\int_0^{\frac{\pi}{6}} v\,\mathrm{d}t$ ***(M1)***

THEN

$= 0.161\left(= \frac{2}{15}\left(1 + e^{-\frac{\pi}{2}}\right)\right)$ ***A1***

[2 marks]

(d) Find the total distance travelled by P in the first 1.5 seconds of its motion. [2]

Markscheme

METHOD 1

EITHER

distance required $= \int_0^{1.5} \left|e^{-3t} \sin 6t\right| dt$ ***(M1)***

OR

distance required $= \int_0^{\frac{\pi}{6}} e^{-3t} \sin 6t \, dt + \left|\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} e^{-3t} \sin 6t \, dt\right| + \int_{\frac{\pi}{3}}^{1.5} e^{-3t} \sin 6t \, dt$ ***(M1)***

$(= 0.16105\ldots + 0.033479\ldots + 0.006806\ldots)$

THEN

$= 0.201 \text{ (m)}$ ***A1***

METHOD 2

using successive minimum and maximum values on the displacement graph ***(M1)***

$0.16105\ldots + (0.16105\ldots - 0.12757\ldots) + (0.13453\ldots - 0.12757\ldots)$

$= 0.201 \text{ (m)}$ ***A1***

[2 marks]

(e) Show that, at these times, $\tan 6t = 2$. [2]

Markscheme

valid attempt to find $\frac{dv}{dt}$ using product rule and set $\frac{dv}{dt} = 0$ ***M1***

$\frac{dv}{dt} = e^{-3t} 6 \cos 6t - 3e^{-3t} \sin 6t$ ***A1***

$\frac{dv}{dt} = 0 \Rightarrow \tan 6t = 2$ ***AG***

[2 marks]

(f) Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -e^{-\frac{\pi}{2}}$. [5]

Markscheme

attempt to evaluate $t_1,\ t_2,\ t_3$ in exact form ***M1***

$6t_1 = \arctan 2 \left(\Rightarrow t_1 = \frac{1}{6}\arctan 2\right)$

$6t_2 = \pi + \arctan 2 \left(\Rightarrow t_2 = \frac{\pi}{6} + \frac{1}{6}\arctan 2\right)$

$6t_3 = 2\pi + \arctan 2 \left(\Rightarrow t_3 = \frac{\pi}{3} + \frac{1}{6}\arctan 2\right)$ ***A1***

Note: The ***A1*** is for any two consecutive correct, or showing that $6t_2 = \pi + 6t_1$ or $6t_3 = \pi + 6t_2$.

showing that $\sin 6t_{n+1} = -\sin 6t_n$

eg $\tan 6t = 2 \Rightarrow \sin 6t = \pm\frac{2}{\sqrt{5}}$ ***M1A1***

showing that $\frac{e^{-3t_{n+1}}}{e^{-3t_n}} = e^{-\frac{\pi}{2}}$ ***M1***

eg $e^{-3\left(\frac{\pi}{6}+k\right)} \div e^{-3k} = e^{-\frac{\pi}{2}}$

Note: Award the ***A1*** for any two consecutive terms.

$\frac{v_3}{v_2} = \frac{v_2}{v_1} = -e^{-\frac{\pi}{2}}$ ***AG***

[5 marks]

8. [Maximum mark: 14]

A rocket is travelling in a straight line, with an initial velocity of 140 m s^{-1}. It accelerates to a new velocity of 500 m s^{-1} in two stages.

During the first stage its acceleration, a m s^{-2}, after t seconds is given by $a(t) = 240\sin(2t)$, where $0 \leqslant t \leqslant k$.

The first stage continues for k seconds until the velocity of the rocket reaches 375 m s^{-1}.

(a) Find an expression for the velocity, v m s^{-1}, of the rocket during the first stage. [4]

Markscheme

recognizing that $v = \int a$ ***(M1)*** correct integration *A1*

eg $-120\cos(2t)+c$

attempt to find c using their $v(t)$ *(M1)*

eg $-120\cos(0)+c=140$

$v(t)=-120\cos(2t)+260$ *A1 N3*

[4 marks]

(b) Find the distance that the rocket travels during the first stage. [4]

Markscheme

evidence of valid approach to find time taken in first stage *(M1)*

eg graph, $-120\cos(2t)+260=375$

$k=1.42595$ *A1*

attempt to substitute **their** v and/or **their** limits into distance formula *(M1)*

eg $\int_0^{1.42595}|v|$, $\int 260-120\cos(2t)$, $\int_0^k(260-120\cos(2t))\,\mathrm{d}t$

353.608

distance is 354 (m) *A1 N3*

[4 marks]

(c) During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage.

Find the total time taken for the two stages. [6]

Markscheme

recognizing velocity of second stage is linear (seen anywhere) *R1*

eg graph, $s=\frac{1}{2}h(a+b)$, $v=mt+c$

valid approach *(M1)*

eg $\int v=353.608$

correct equation *(A1)*

eg $\frac{1}{2}h(375+500)=353.608$

time for stage two $=0.808248$ (0.809142 from 3 sf) *A2*

2.23420 (2.23914 from 3 sf)

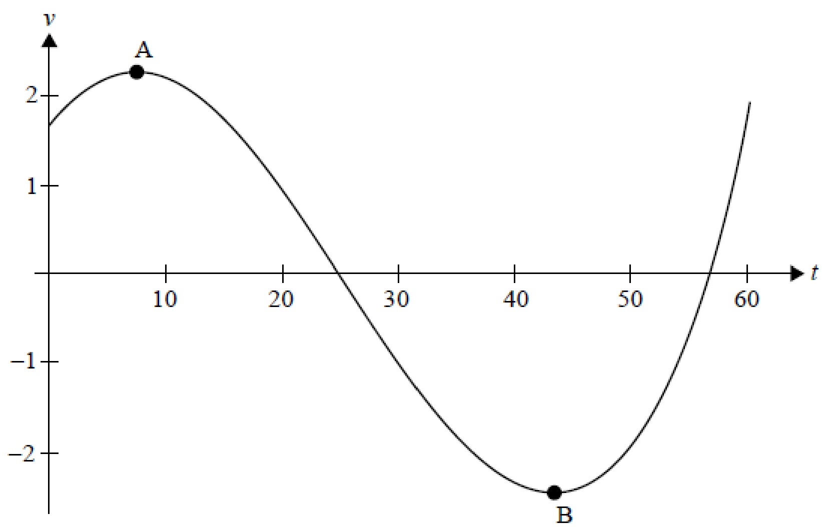
2.23 seconds (2.24 from 3 sf) ***A1 N3***

[6 marks]

9. [Maximum mark: 14]

A body moves in a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, after t seconds is given by $v = 2\sin\left(\frac{t}{10} + \frac{\pi}{5}\right)\csc\left(\frac{t}{30} + \frac{\pi}{4}\right)$ for $0 \leqslant t \leqslant 60$.

The following diagram shows the graph of v against t. Point A is a local maximum and point B is a local minimum.



The body first comes to rest at time $t = t_1$. Find

(a) Determine the coordinates of point A and the coordinates of point B. [4]

Markscheme

$\mathrm{A}\,(7.47,\ 2.28)$ and $\mathrm{B}\,(43.4,\ -2.45)$ ***A1A1A1A1***

[4 marks]

(b) Hence, write down the maximum speed of the body. [1]

Markscheme

maximum speed is $2.45\ \left(\mathrm{ms}^{-1}\right)$ ***A1***

[1 mark]

(c) the value of t_1. [2]

Markscheme

$v = 0 \Rightarrow t_1 = 25.1\ (\text{s})$ ***(M1)A1***

[2 marks]

(d) the distance travelled between $t = 0$ and $t = t_1$. [2]

Markscheme

$\int_0^{t_1} v\,\text{d}t$ ***(M1)***

$= 41.0\ (\text{m})$ ***A1***

[2 marks]

(e) the acceleration when $t = t_1$. [2]

Markscheme

$a = \frac{\text{d}v}{\text{d}t}$ at $t = t_1 = 25.1$ ***(M1)***

$a = -0.200\ \left(\text{ms}^{-2}\right)$ ***A1***

Note: Accept $a = -0.2$.

[2 marks]

(f) Find the distance travelled in the first 30 seconds. [3]

Markscheme

attempt to integrate between 0 and 30 ***(M1)***

Note: An unsupported answer of 38.6 can imply integrating from 0 to 30.

EITHER

$\int_0^{30} |v|\,\text{d}t$ ***(A1)***

OR

$41.0 - \int_{t_1}^{30} v\,\text{d}t$ ***(A1)***

THEN

$= 43.3\ (\text{m})$ ***A1***

[3 marks]