Sequences 1 [41 marks]

1. [Maximum mark: 6] 19N.1.SL.TZ0.S_1

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

Markscheme

valid approach ***(M1)***

eg $11 - 5, 11 = 5 + d$

$d = 6$ ***A1 N2***

[2 marks]

(b) Find the first term. [2]

Markscheme

valid approach ***(M1)***

eg $u_2 - d$, $5 - 6$, $u_1 + (3 - 1)(6) = 11$

$u_1 = -1$ ***A1 N2***

[2 marks]

(c) Find the sum of the first 20 terms. [2]

Markscheme

correct substitution into sum formula

eg $\frac{20}{2}(2(-1) + 19(6))$, $\frac{20}{2}(-1 + 113)$ ***(A1)***

$S_{20} = 1120$ **A1 N2**

[2 marks]

2. [Maximum mark: 5] 19N.2.AHL.TZ0.H_1

A geometric sequence has $u_4 = -70$ and $u_7 = 8.75$. Find the second term of the sequence. [5]

Markscheme

$u_1 r^3 = -70$, $u_1 r^6 = 8.75$ **(M1)**

$r^3 = \frac{8.75}{-70} = -0.125$ **(A1)**

$\Rightarrow r = -0.5$ **(A1)**

valid attempt to find u_2 **(M1)**

for example: $u_1 = \frac{-70}{-0.125} = 560$

$u_2 = 560 \times -0.5$

$= -280$ **A1**

[5 marks]

3. [Maximum mark: 6] 24N.1.SL.TZ1.6

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$. [6]

Markscheme

attempt to use $u_n = u_1 + (n-1)d$ or
$S_n = \frac{n}{2}[2u_1 + (n-1)d]$ or $S_n = \frac{n}{2}[u_1 + u_n]$ to set up at least one equation in u_1 and d ***(M1)***

$14 = u_1 + 9d$ and $200 = \frac{25}{2}[2u_1 + 24d]$ ***A1***

attempt to solve their two linear equations in u_1 and d simultaneously (must eliminate one variable) ***(M1)***

$d = -2 \ (\Rightarrow u_1 = 32)$ ***(A1)***

attempt to solve $u_k = 0$ with their d (or with their d and u_1) ***(M1)***

$\Rightarrow k = 17$ ***A1***

[6 marks]

4. [Maximum mark: 14] 23M.1.SL.TZ1.8

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

Markscheme

recognition that $n = 5$ ***(M1)***

$S_5 = 45$ ***A1***

[2 marks]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

Markscheme

METHOD 1

recognition that $S_5 + u_6 = S_6$ ***(M1)***

$u_6 = 15$ ***A1***

METHOD 2

recognition that $60 = \frac{6}{2}(S_1 + u_6)$ ***(M1)***

$60 = 3(5 + u_6)$

$u_6 = 15$ ***A1***

METHOD 3

substituting their u_1 and d values into $u_1 + (n-1)d$ ***(M1)***

$u_6 = 15$ ***A1***

[2 marks]

(b) Find u_1. [2]

Markscheme

recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their u_6 into S_6 ***(M1)***

OR equations for S_5 and S_6 in terms of u_1 and d

$1 + 4$ OR $60 = \frac{6}{2}\left(U_1 + 15\right)$

$u_1 = 5$ ***A1***

[2 marks]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Markscheme

EITHER

valid attempt to find d (may be seen in (a) or (b)) ***(M1)***

$d = 2$ ***(A1)***

OR

valid attempt to find $S_n - S_{n-1}$ ***(M1)***

$n^2 + 4n - \left(n^2 - 2n + 1 + 4n - 4\right)$ ***(A1)***

OR

equating $n^2 + 4n = \frac{n}{2}\left(5 + u_n\right)$ ***(M1)***

$2n + 8 = 5 + u_n$ (or equivalent) ***(A1)***

THEN

$u_n = 5 + 2(n - 1)$ OR $u_n = 2n + 3$ ***A1***

[3 marks]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

Markscheme

recognition that $v_2 r^2 = v_4$ OR $(v_3)^2 = v_2 \times v_4$ ***(M1)***

$r^2 = 3$ OR $v_3 = \left(\pm\right) 5\sqrt{3}$ ***(A1)***

$r = \pm\sqrt{3}$ ***A1***

Note: If no working shown, award ***M1A1A0*** for $\sqrt{3}$.

[3 marks]

(e) Given that $v_{99} < 0$, find v_5. [2]

Markscheme

recognition that r is negative ***(M1)***

$v_5 = -15\sqrt{3}$ $\left(= -\frac{45}{\sqrt{3}}\right)$ ***A1***

[2 marks]

5. [Maximum mark: 5] 21M.1.SL.TZ1.3

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d. [5]

Markscheme

METHOD 1 (finding u_1 first, from S_8)

$4(u_1 + 8) = 8$ ***(A1)***

$u_1 = -6$ ***A1***

$u_1 + 7d = 8$ OR $4(2u_1 + 7d) = 8$ (may be seen with their value of u_1) ***(A1)***

attempt to substitute their u_1 ***(M1)***

$d = 2$ ***A1***

METHOD 2 (solving simultaneously)

$u_1 + 7d = 8$ ***(A1)***

$4(u_1 + 8) = 8$ OR $4(2u_1 + 7d) = 8$ OR $u_1 = -3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1 = -6,\ d = 2$ ***A1A1***

[5 marks]

6. [Maximum mark: 5] 21M.2.SL.TZ2.3

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Markscheme

attempt to use $u_1 + (n-1)d = 0$ ***(M1)***

$60 - 2.5(k-1) = 0$

$k = 25$ ***A1***

[2 marks]

(b) Let S_n denote the sum of the first n terms of the sequence.

Find the maximum value of S_n. [3]

Markscheme

METHOD 1

attempting to express S_n in terms of n ***(M1)***

use of a graph or a table to attempt to find the maximum sum ***(M1)***

$= 750$ ***A1***

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ ***(M1)***

$S_{25} = \frac{25}{2}(60 + 0),\ S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5)$ ***(A1)***

OR

attempting to calculate S_{24} ***(M1)***

$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5)$ ***(A1)***

THEN

$= 750$ ***A1***

[3 marks]