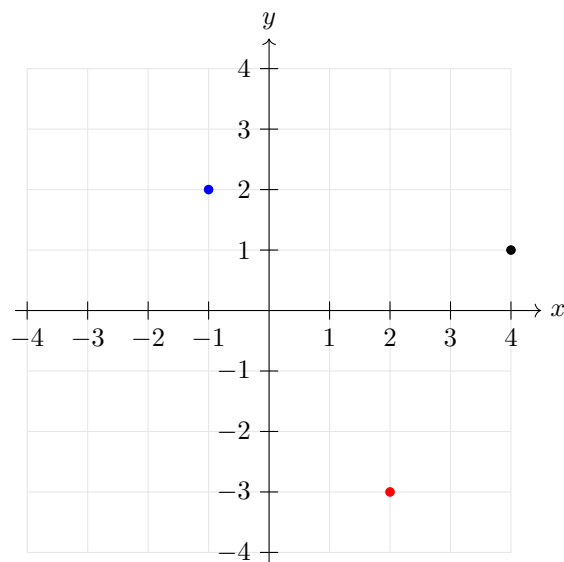


Chapter 1

Coordinate geometry

1.1 Coordinate system

A point in a 2D coordinate system (coordinate plane) can be uniquely identified by 2 coordinates. We usually refer to the first coordinate as the x -coordinate and the second coordinate as the y -coordinate. For instance the point $(4, 1)$ has x -coordinate equal to 4 and y -coordinate equal to 1. It is represented with a black point below.



The blue and red points represent $(-1, 2)$ and $(2, -3)$ respectively.

Exercise 3.1.1a Label the following points on a 2d coordinate system:

(a) $(1, 1)$

(b) $(-2, -3)$

(c) $(3, -4)$

(d) $(0, 4)$

(e) $(-3, 0)$

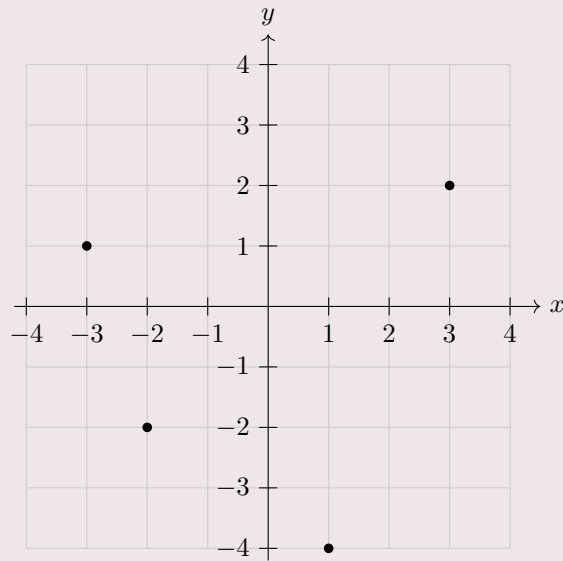
(f) $(0, 0)$

(g) $(\pi, 2)$

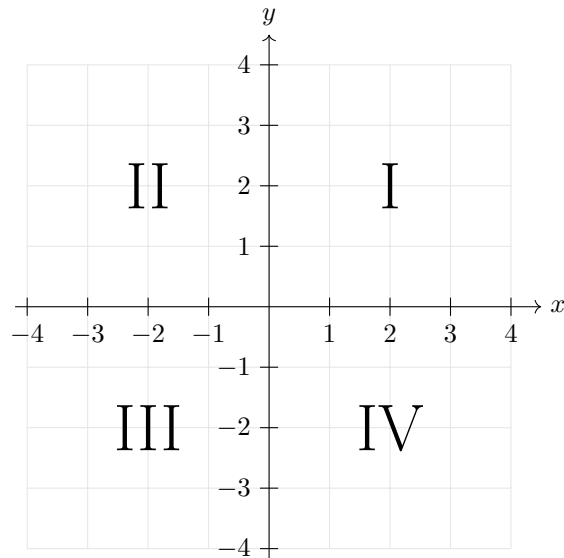
(h) $(\sqrt{2}, -2\sqrt{2})$

(i) $(5 - 2\sqrt{6}, 3\sqrt{3} - 5)$

Exercise 3.1.1b Write down the coordinates of the following points (all coordinates are integers):



A 2D coordinate plane can be divided into 4 quadrants (usually denoted with Roman numerals):



In the first quadrant both coordinates are positive. In the second quadrant we have $x < 0$ and $y > 0$. In quadrant III: $x < 0$ and $y < 0$. Finally in quadrant IV: $x > 0$ and $y < 0$.

Exercise 3.1.1c State in which quadrant the following points are located:

- | | | |
|------------------------|---------------------------|---|
| (a) $(2, -3)$ | (b) $(-1, -20)$ | (c) $(2 - \sqrt{3}, 4\sqrt{5} - 9)$ |
| (d) $((-2)^3, (-2)^4)$ | (b) $(3^0, \sqrt{7} - 3)$ | (c) $(1 - \sqrt{5} , 2^{\sqrt[3]{3}} - 5)$ |

Note that the axes are in general not considered to be parts of any quadrant. So for example the point $(0, 3)$ lies on the y -axis (specifically the positive part of the y -axis) and not in any of the quadrants.

Exercise 3.1.2a In a 2D video game, a character moves on a 10×10 coordinate grid. The character begins at the spawn point $(0, 0)$.

(a) The character moves to collect a power-up at $(3, -1)$, then to a checkpoint at $(-2, -4)$. Plot and label these three points on a coordinate plane.

(b) To unlock a secret level, the character must reach a point where $x + y = 3$. Find two points with integer coordinates that satisfy this equation and plot them.

(c) The game restricts the character from entering Quadrant IV. Which of the points from parts (a) and (b) are in Quadrant IV, if any?

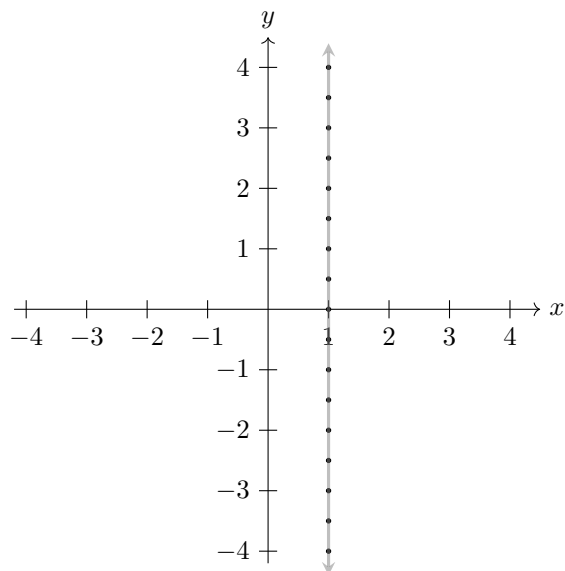
Exercise 3.1.2b A smart vacuum cleaner navigates a room on an 8×8 coordinate grid. It starts at its docking station at $(0, 0)$.

(a) The vacuum moves to clean a spill at $(1, 3)$, then to a pet bed at $(2, -2)$. Plot and label these three points on a coordinate plane.

(b) To avoid obstacles, the vacuum must stop at a point where $y = 2$. List three points with integer x -coordinates that satisfy this equation and plot them.

(c) If the vacuum is programmed to stay out of Quadrant I, which of the points from parts (a) and (b) should it avoid?

Suppose we would like to label all points with x -coordinate equal to 1. These points include $(1, -2)$, $(1, 0)$ and $(1, \sqrt{5})$. Of course there are infinitely many points with the first coordinate equal to 1. We can label some of them on the coordinate plane as follows:



Note that the diagram includes 17 points that have the first coordinate equal to 1, that is they satisfy the equation $x = 1$. The diagram also include a line that indicates **all** points satisfying the equation $x = 1$. Note that the line extends indefinitely as indicated by the arrows.

Exercise 3.1.3 On a 2d coordinate system label all points that satisfy the following equations:

(a) $x = -2$

(b) $y = 4$

(c) $2x = 6$

(d) $x = y$

(e) $x + y = 2$

(f) $x - y = 1$

(g) $2x + y = 3$

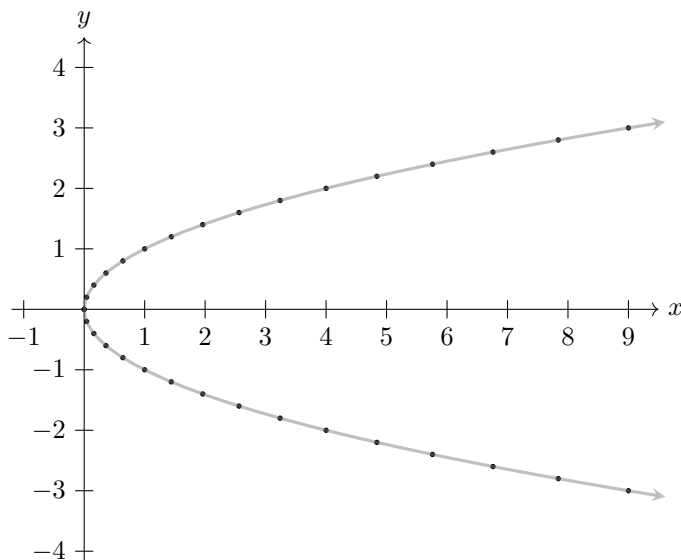
(h) $3x - 2y = 6$

(i) $\frac{x}{2} - y = 2$

Consider the equation $x = y^2$. Points whose first coordinate is the square of the second coordinate satisfy this equation. Examples of these points include $(1, 1)$, $(4, 2)$, $(9, -3)$. These points satisfy the given equation because $1 = 1^2$, $4 = 2^2$ and $9 = (-3)^2$. Of course there are many more points that satisfy the equation. We can create a table of these points by varying the possible values of the second coordinate:

x	9	7.84	6.76	...	0.04	0	0.04	...	4	4.84	5.76	...
y	-3	-2.8	-2.6	...	-0.2	0	0.2	...	2	2.2	2.4	...

We can mark such points on the coordinate plane:



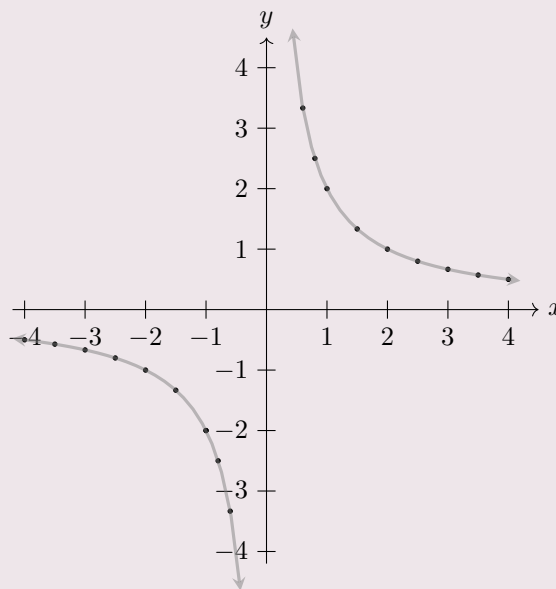
Note that the above diagram includes 31 points that satisfy the equation $x = y^2$, these are, of course, not all points that satisfy the equation. The gray curve includes all points that satisfy the equation. Note that the curve extends indefinitely as indicated by arrows.

Worked example 3.1.4.

Label several points on the coordinate plane that satisfy the equation $xy = 2$ and hence draw a curve that contains all points that satisfy this equation.

Examples of points satisfying $xy = 2$ include $(2, 1)$, $(1, 2)$, but also $(4, \frac{1}{2})$, $(-3, -\frac{2}{3})$ and $(-1, -2)$. Note that the point $(2, 1)$ satisfies the equation $xy = 2$, because $2 \times 1 = 2$. We can include more points by varying one of the coordinates:

x	4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4	...
y	$\frac{1}{2}$	$-\frac{2}{3}$	-1	-2	-4	-	4	2	1	$\frac{2}{3}$	$\frac{1}{2}$...



The diagram below includes several points satisfying the equation $xy = 2$ and also a curve that indicates all points satisfying this equation. Note that no points with x -coordinate or y -coordinate 0 satisfies this equation.

Exercise 3.1.4 On a 2d coordinate system label all points that satisfy the following equations:

(a) $y = x^2$

(b) $y = \sqrt{x}$

(c) $y = -\sqrt{x}$

(d) $y = 2^x$

(e) $x = 2^y$

(f) $y = 3^x$

(g) $y = |x|$

(h) $x = |y|$

(i) $x^2 = y^2$

Worked example 3.1.5.

Label all points on the coordinate plane that satisfy the equation:

$$x^2 + 2x - 2y = xy$$

We start by moving all terms to one side:

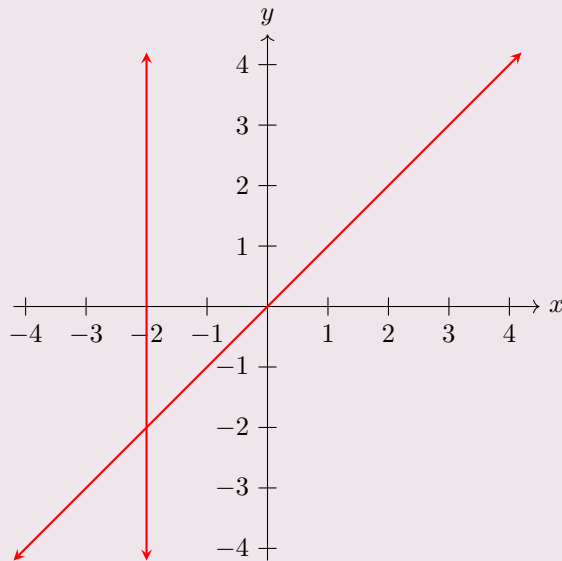
$$x^2 + 2x - 2y - xy = 0$$

We can now factorize the left hand side by grouping:

$$x(x + 2) - y(2 + x) = 0$$

$$(x - y)(x + 2) = 0$$

So either $x = y$ or $x = -2$. We can now draw all the points that satisfy the above equations:



Any point on either of the two lines satisfies the given equation.

Exercise 3.1.5. On a 2d coordinate system label all points that satisfy the following equations:

(a) $xy + 3x - y = 3$

(b) $xy + y^2 - 2x = 2y$

(c) $x^3 + y = x^2 + yx$

(d) $x^2 + xy^2 + 3y^2 + 3x = 0$

(e) $2^{x+y} - x2^x - y2^y + xy = 0$

(f) $x^22^x - y^2 = yx^2 - y2^x$

SHORT TEST

1. [2 *points*]

State in which quadrant the following points are located:

(a) $(4 - \sqrt{17}, (-2)^3)$

(b) $(|2 - \sqrt{11}|, -2^2)$

2. [6 *points*]

A smart vacuum cleaner navigates a room on an 8×8 coordinate grid. It starts at its docking station at $(1, 1)$.

- (a) The vacuum moves to clean a spill at $(3, 4)$, then to a toy at $(0, -1)$. Plot and label these three points on a coordinate plane.
- (b) To avoid furniture, the vacuum must stop at a point where $x + y = 3$. List three points with integer x -coordinates that satisfy this equation and plot them.
- (c) If the vacuum is programmed to stay out of Quadrant IV, which of the points from parts (a) and (b) should it avoid?

3. [4 *points*]

Sketch the set of points satisfying the following equations:

(a) $x + y = 1$

(b) $y = x^2 - 1$

**SHORT TEST
SOLUTIONS**

- 1.** [2 points]
State in which quadrant the following points are located:

(a) $(4 - \sqrt{17}, (-2)^3)$ (b) $(|2 - \sqrt{11}|, -2^2)$

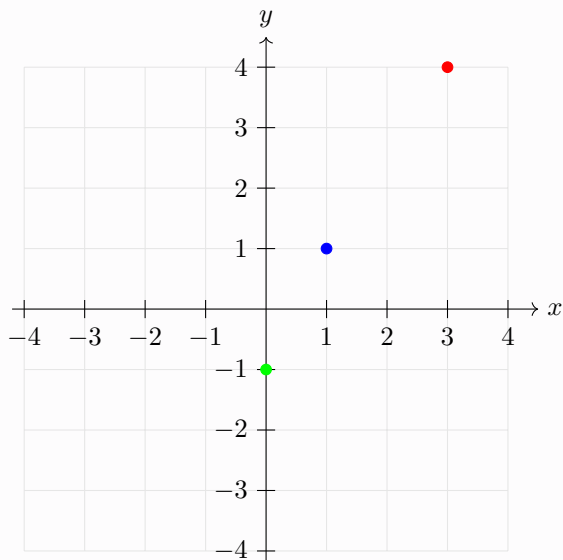
(a) $4 - \sqrt{17} < 0$ and $(-2)^3 = -8 < 0$, so III quadrant.

(b) $|2 - \sqrt{11}| > 0$ and $-2^2 = -4 < 0$, so IV quadrant.

- 2.** [6 points]
A smart vacuum cleaner navigates a room on an 8×8 coordinate grid. It starts at its docking station at $(1, 1)$.

- (a) The vacuum moves to clean a spill at $(3, 4)$, then to a toy at $(0, -1)$. Plot these three points on a coordinate plane.
- (b) To avoid furniture, the vacuum must stop at a point where $x + y = 3$. List three points with integer x -coordinates that satisfy this equation and plot them.
- (c) If the vacuum is programmed to stay out of Quadrant IV, which of the points from parts (a) and (b) should it avoid?

(a)



blue - docking station, red - spill, green - toy.

(b) For example $(1, 2)$, $(3, 0)$ and $(4, -1)$.

(c) In the IV quadrant we have $x > 0$ and $y < 0$, so only $(4, -1)$ is be avoided.

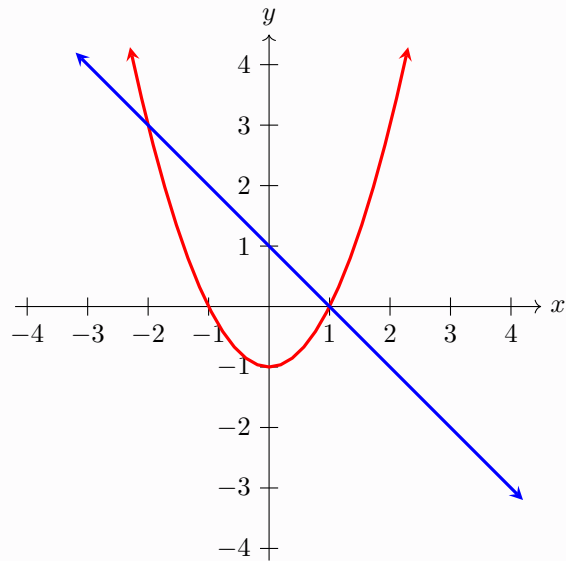
3.

[4 points]

Sketch the set of points satisfying the following equations:

(a) $x + y = 1$

(b) $y = x^2 - 1$

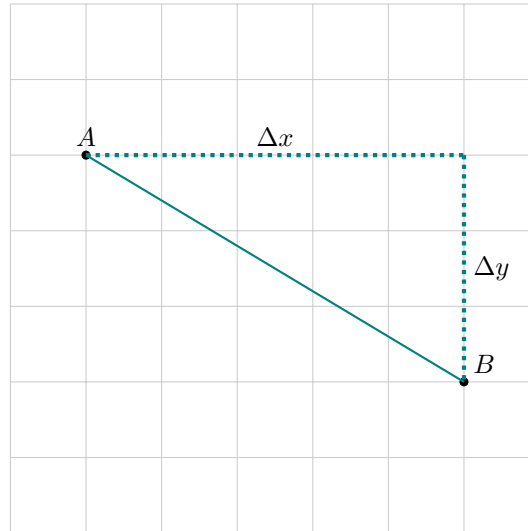


(a) in blue, (b) in red.

1.2 Line segments

A line segment between points A and B is denoted as \overline{AB} . Length of a line segment \overline{AB} , i.e. the distance between points $A(x_A, y_A)$ and $B(x_B, y_B)$, can be calculated using Pythagora's theorem:

$$|AB| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$



Note that Δx and Δy denote the change in x and y -coordinates, respectively. Δx is the horizontal change and Δy is the vertical change.

Worked example 3.2.1.

Find the length of the line segment \overline{AB} , where $A = (-1, -3)$ and $B = (4, -1)$.

The x -coordinate changes from -1 to 4 , so the change in the x -coordinate is:

$$\Delta x = 4 - (-1) = 5$$

The y -coordinate changes from -3 to -1 , so the change in the y -coordinate is:

$$\Delta y = -1 - (-3) = 2$$

So the distance from A to B (the length of \overline{AB}) is:

$$|AB| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

Exercise 3.2.1. Find the length of the line segments \overline{AB} , where:

(a) $A = (-2, 1)$ $B = (-3, 5)$

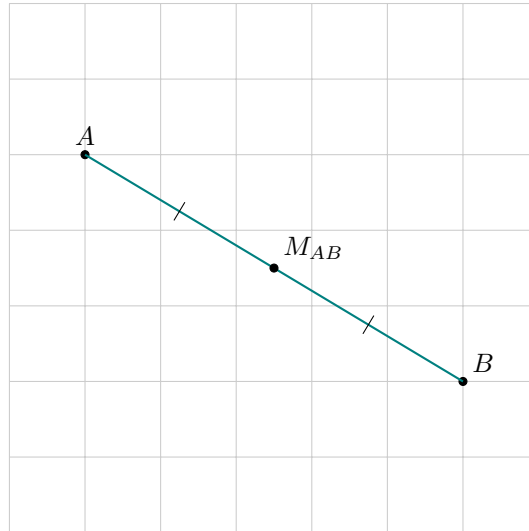
(b) $A = (3, 4)$ $B = (2, 0)$

(c) $A = (0, 5)$ $B = (\sqrt{2}, 1)$

(d) $A = (a, 2a)$ $B = (2a - 1, a + 2)$

The **midpoint** of line segment \overline{AB} is a point on \overline{AB} , which is equidistant from A and B . For $A(x_A, y_A)$ and $B(x_B, y_B)$ the midpoint of \overline{AB} is:

$$M_{AB} = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right)$$



Worked example 3.2.2.

Find the midpoint of the line segment \overline{AB} , where $A(-2, 4)$ and $B(4, 0)$

We have:

$$M_{AB} = \left(\frac{-2 + 4}{2}, \frac{4 + 0}{2} \right) = (1, 2)$$

Exercise 3.2.2a Find the midpoint of the line segments \overline{AB} , where:

(a) $A = (-6, 1)$ $B = (2, 3)$

(b) $A = (\sqrt{2}, \sqrt{3})$ $B = (3\sqrt{2}, -5\sqrt{3})$

(c) $A = (t, 3t + 1)$ $B = (1 - t, t - 3)$

(d) $A = (a + b, a - b)$ $B = (3a - 5b, b - 3a)$

Exercise 3.2.2b Find the coordinate of point A given that:

(a) $B = (-3, 2)$ $M_{AB} = (-1, 1)$

(b) $B = (0, 5)$ $M_{AB} = (-1, 2)$

(c) $B = (2p, p - 1)$ $M_{AB} = (p, 3p)$

(d) $B = (t + 2, 3 - t)$ $M_{AB} = (t - 1, 2t)$

Worked example 3.2.3.

The line segment \overline{AB} with $A(2, k)$ has a midpoint $M_{AB} = (5, 3)$. Given that the length of \overline{AB} is $\sqrt{40}$, find the possible values of k and the coordinate of B .

Using the midpoint formula we get that:

$$\frac{2 + x_B}{2} = 5 \quad \text{and} \quad \frac{k + y_B}{2} = 3$$

Which gives the coordinates of B as $(8, 6 - k)$. Now we have:

$$\Delta x = 6 \quad \text{and} \quad \Delta y = 6 - 2k$$

So using the length of \overline{AB} we get:

$$\sqrt{6^2 + (6 - 2k)^2} = \sqrt{40}$$

Which gives:

$$(6 - 2k)^2 = 4$$

So:

$$6 - 2k = \pm 2$$

And finally we get $k = 2$ or $k = 4$, so B has coordinates $(8, 4)$ or $(8, 2)$.

Exercise 3.2.3a The line segment \overline{AB} has endpoints $A(-1, 3)$ and $B(4, b)$. If the length of AB is $\sqrt{41}$, find the possible coordinates of B .

Exercise 3.2.3b The line segment \overline{AB} has endpoints $A(1, -2)$ and $B(5, t + 2)$. If the coordinates of the midpoint are $M(3, t - 1)$, find the value of t and length of \overline{AB} .

Exercise 3.2.3c The line segment \overline{AB} has endpoint $A(-2, k)$ and midpoint $M(-3, -1)$. If the length of AB is $2\sqrt{5}$, find the possible values of k and the coordinates of B .

Exercise 3.2.3d The line segment \overline{AB} has endpoint $A(2, 1)$ and midpoint $M(q, 2)$. If the length of AB is $2\sqrt{10}$, find the possible values of q and the coordinates of B .

Exercise 3.2.3e The line segment \overline{AB} has endpoint $A(a, 1)$ and midpoint $M(2, a)$. If the length of AB is $2\sqrt{5}$, find the possible values of a and the coordinates of B .

Worked example 3.2.4.

The line segment \overline{AB} has endpoint $A(1, 5)$ and midpoint $M(2, k)$. The coordinates of point B satisfy the equation $x + y = 0$. Find the value of k and the length of \overline{AB} .

Using the midpoint we must have that:

$$\frac{1 + x_B}{2} = 2$$

which gives $x_B = 3$, so B has coordinates $(3, y_B)$. Now they must satisfy the equation $x + y = 0$, so:

$$3 + y_B = 0$$

which gives $y_B = -3$ and the coordinates of B are $(3, -3)$. We can now calculate the y -coordinate of the midpoint:

$$k = \frac{5 + (-3)}{2} = 1$$

We can also calculate the length of \overline{AB} . We have $\Delta x = 2$ and $\Delta y = 8$, so:

$$|AB| = \sqrt{2^2 + 8^2} = \sqrt{68} = 2\sqrt{17}$$

Exercise 3.2.4a The line segment \overline{AB} has endpoint $A(2, 1)$ and midpoint $M(3, m)$. The coordinates of point B satisfy the equation $x + y = 2$. Find the value of m and the length of \overline{AB} .

Exercise 3.2.4b The line segment \overline{AB} has endpoint $A(0, 2)$ and midpoint $M(q, 5)$. The coordinates of point B satisfy the equation $2x + y = 4$. Find the value of q and the length of \overline{AB} .

Exercise 3.2.4c The line segment \overline{AB} has endpoint $A(-1, 1)$ and midpoint $M(2, r)$. The coordinates of point B satisfy the equation $y = x^2$. Find the value of r and the length of \overline{AB} .

Exercise 3.2.4d The line segment \overline{AB} has endpoint $A(\sqrt{2}, \sqrt{3})$ and midpoint $M(2\sqrt{2}, s)$. The coordinates of point B satisfy the equation $3y^2 = x^2$. Find the possible values s .

Exercise 3.2.4e The line segment \overline{AB} has endpoint $B(1, -2)$ and midpoint $M(t, 1)$. The coordinates of point A satisfy the equation $xy = 5$. Find the value of t .

The **gradient** of a line segment \overline{AB} is the ratio of the change in y -coordinate (Δy) to the change in x -coordinate (Δx). The gradient is usually denoted with the letter m .

$$m = \frac{\Delta y}{\Delta x} = \frac{y_B - y_A}{x_B - x_A}$$

Note that the gradient can be thought of as *rise* over *run*, where *rise* refers to the vertical change and *run* refers to the horizontal change.

Note that for horizontal line segments the gradient is 0, since $\Delta y = 0$ and for vertical line segments the gradient is undefined, since $\Delta x = 0$.

The gradient of \overline{AB} is of course equal to the gradient of \overline{BA} . This follows from the formula, but also from the fact the line segments have no direction and hence \overline{AB} is the same line segment as \overline{BA} .

Exercise 3.2.5 Find the gradient the line segments \overline{AB} , where:

(a) $A = (1, 5)$ $B = (2, 3)$

(b) $A = (-1, -5)$ $B = (3, 1)$

(c) $A = (2, 5)$ $B = (4, 5)$

(d) $A = (1, -5)$ $B = (1, 4)$

Worked example 3.2.6

The line segment \overline{AB} has endpoints $A(-1, 3)$ and $B(3, b)$. Find b , given that the gradient of this line segment is $-\frac{1}{2}$.

We have $\Delta x = 3 - (-1) = 4$ and $\Delta y = b - 3$, so, using the gradient, we get:

$$\frac{b - 3}{4} = -\frac{1}{2}$$

which gives $b = 1$.

Exercise 3.2.6 Find the unknown coordinates in the following examples:

(a) $A = (-1, -2)$ $B = (3, a)$ $m = 1$

(b) $A = (4, 1)$ $B = (b, 2)$ $m = \frac{1}{4}$

(c) $A = (c, 3)$ $B = (3, 4)$ $m = 2$

(d) $A = \left(\frac{1}{2}, -\frac{1}{3}\right)$ $B = (1, d)$ $m = 3$

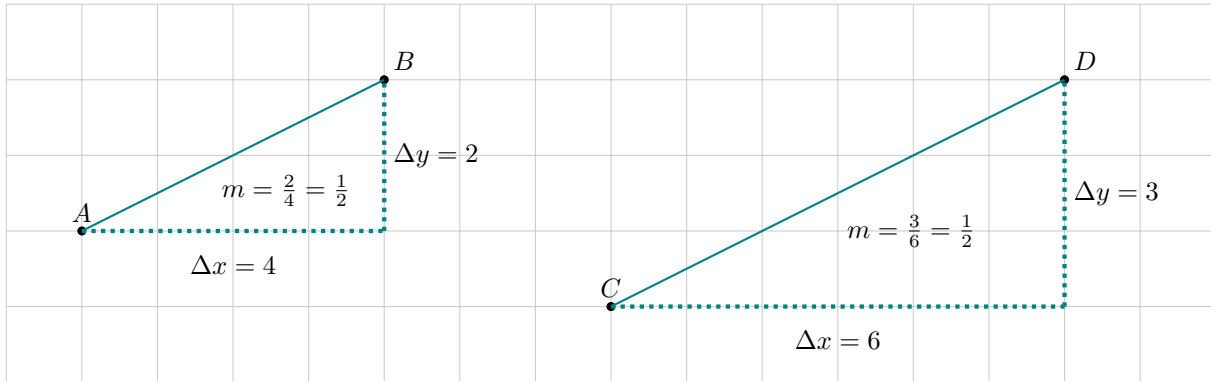
(e) $A = (e, 2e)$ $B = (3, 4)$ $m = 3$

(f) $A = (-1, f)$ $B = (2f, 3)$ $m = -1$

(g) $A = (1, 1)$ $B = (g, g^2)$ $m = 3$

(h) $A = (1, 1)$ $B = \left(h, \frac{1}{h}\right)$ $m = -\frac{1}{2}$

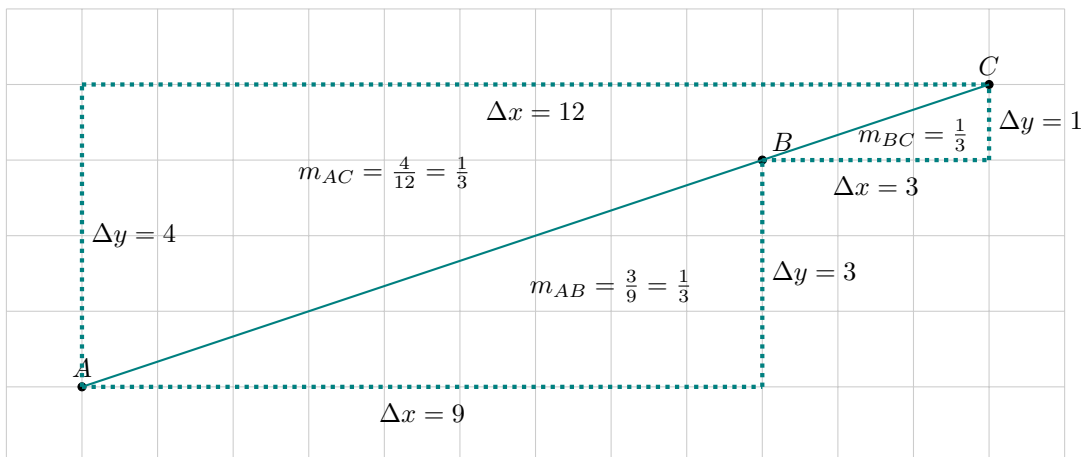
Two line segments are parallel, if their gradients are the same:



Exercise 3.2.7. Find unknown coordinates given that the line segments \overline{AB} and \overline{CD} are parallel.

- (a) $A(-2, 3)$, $B(1, 4)$, $C(-5, 1)$, $D(1, d)$ (b) $A(0, -2)$, $B(2, -6)$, $C(c, 7)$, $D(1, 1)$
 (c) $A(a, 0)$, $B(2, 3)$, $C(-2, -1)$, $D(\sqrt{2}, \sqrt{2})$ (d) $A(3, 1)$, $B(\sqrt{3}, b)$, $C(1, 2)$, $D(\sqrt{3}, 3)$

Three points A, B and C are collinear (lie on the same line, or there is a line segment containing all three), if the gradient of \overline{AB} is equal to the gradient of \overline{AC} (it is also equal to the gradient of \overline{BC}).



Exercise 3.2.8. Find unknown coordinates given that the points A, B and C are collinear.

- (a) $A(0, 1)$, $B(3, 7)$, $C(4, p)$ (b) $A(-2, -4)$, $B(1, q)$, $C(2, -2)$
 (c) $A(r, 0)$, $B(3, 5)$, $C(-3, -10)$ (d) $A(\sqrt{2}, 2\sqrt{2})$, $B(5\sqrt{2}, \sqrt{2})$, $C(4\sqrt{2}, s)$

Worked example 3.2.9.

The line segment \overline{AB} has endpoint $A(-1, 4)$. Find the coordinates of B , given that the gradient of this line segment is -1 and the length \overline{AB} is $3\sqrt{2}$.

We have $\Delta x = x_B + 1$ and $\Delta y = y_B - 4$. Using the gradient formula we get that:

$$\frac{y_B - 4}{x_B + 1} = -1$$

which gives $y_B = 3 - x_B$. Now using the distance formula:

$$\sqrt{(x_B + 1)^2 + (y_B - 4)^2} = 3\sqrt{2}$$

$$\sqrt{(x_B + 1)^2 + (3 - x_B - 4)^2} = 3\sqrt{2}$$

$$2(x_B + 1)^2 = 18$$

$$x_B + 1 = \pm 3$$

So either $x_B = 2$ and then $B = (2, 1)$ or $x_B = -4$ and $B = (-4, 7)$.

Exercise 3.2.9a The line segment \overline{AB} has endpoint $A(2, 1)$. Find the coordinates of B , given that the gradient of this line segment is 2 and the length \overline{AB} is $\sqrt{5}$.

Exercise 3.2.9b The line segment \overline{AB} has endpoint $A(0, 3)$. Find the coordinates of B , given that the gradient of this line segment is -2 and the length \overline{AB} is $2\sqrt{5}$.

Exercise 3.2.9c The line segment \overline{AB} has endpoint $A(-2, 0)$. Find the coordinates of B , given that the gradient of this line segment is $\frac{1}{2}$ and the coordinates of points B satisfy the equation $y = x$.

Exercise 3.2.9d The line segment \overline{AB} has endpoint $A(-3, -1)$. Find the coordinates of B , given that the gradient of this line segment is 2 and the coordinates of points B satisfy the equation $x + y = 2$.

Exercise 3.2.9e The line segment \overline{AB} has endpoint $A(-1, 1)$. Find the coordinates of B , given that the gradient of this line segment is 1 and the coordinates of points B satisfy the equation $y = x^2$.

Equation of a circle

1. Find the distance between $(3, 1)$ and $(6, 5)$.

2. Write down an expression for the distance between $(3, 1)$ and (x, y) .

A circle with centre at C is a set of all points at a given distance from C .

3. Show that the equation for the circle centered at $(3, 1)$ of radius 5 can be written as:

$$(x - 3)^2 + (y - 1)^2 = 5^2$$

4. Find the equation of a circle with the given centre and radius:

(a) $(1, 2)$, $r = 3$

(b) $(0, -3)$, $r = 1$

(c) $(-3, -2)$, $r = \sqrt{5}$

(b) $(-1, 5)$, $r = \frac{1}{2}$

5. Find the centre and the radius of the circles given by the following equations:

(a) $(x - 2)^2 + (y - 7)^2 = 6^2$

(b) $x^2 + y^2 = 9$

(c) $(x + 1)^2 + (y - 4)^2 = 2$

(d) $x^2 + (y - 5)^2 = 7$

6. Find the centre and the radius of the circles given by the following equations:

(a) $x^2 - 2x + 1 + y^2 - 4x + 4 = 25$

(b) $x^2 + 2x + y^2 - 6x = 5$

(c) $x^2 + 4x + y^2 - 8y + 19 = 0$

(d) $x^2 + x + y^2 - 3y = 0$

SHORT TEST

1. [3 points]

For the line segment \overline{AB} with endpoints $A(-1, -4)$ and $B(3, 2)$, find:

- (a) midpoint, (b) length, (c) gradient.

2. [5 points]

Let $A(-2, 0)$, $B(1, 2)$ and $C(c, 3)$. Find c , if:

- (a) A , B and C are collinear.
(b) The midpoint of AC has coordinates $(4, 1.5)$.
(c) $|AB| = |AC|$.

3. [4 points]

The line segment \overline{AB} has endpoint $A(1, -2)$. Find the coordinates of B , given that the gradient of this line segment is 3 and the length \overline{AB} is $\sqrt{10}$.

**SHORT TEST
SOLUTIONS**

1. For the line segment \overline{AB} with endpoints $A(-1, -4)$ and $B(3, 2)$, find: [3 points]

- (a) midpoint, (b) length, (c) gradient.

$$(a) M_{AB} = \left(\frac{-1+3}{2}, \frac{-4+2}{2} \right) = (1, -1).$$

$$(b) |AB| = \sqrt{(3 - (-1))^2 + (2 - (-4))^2} = 2\sqrt{13}$$

$$(c) m_{AB} = \frac{2 - (-4)}{3 - (-1)} = \frac{3}{2}$$

2. Let $A(-2, 0)$, $B(1, 2)$ and $C(c, 3)$. Find c , if: [5 points]

- (a) A, B and C are collinear.
 (b) The midpoint of AC has coordinates $(4, 1.5)$.
 (c) $|AB| = |AC|$.

$$(a) m_{AB} = \frac{2}{3}, \text{ so } \frac{2}{3} = \frac{3}{c+2}, \text{ which gives } c = 2.5.$$

$$(b) \frac{-2+c}{2} = 4, \text{ so } c = 10$$

$$(c) |AB| = \sqrt{3^2 + 2^2} = \sqrt{13}, \text{ so}$$

$$\sqrt{13} = \sqrt{(c+2)^2 + 9}$$

which gives $(c+2)^2 = 4$, so $c+2 = \pm 2$ and so $c = 0$ or $c = -4$.

3. The line segment \overline{AB} has endpoint $A(1, -2)$. Find the coordinates of B , given that the gradient of this line segment is 3 and the length \overline{AB} is $\sqrt{10}$. [4 points]

We have $\Delta x = x_B - 1$ and $\Delta y = y_B + 2$. Using the gradient we have:

$$\frac{y_B + 2}{x_B - 1} = 3$$

which gives $y_B = 3x_B - 5$. Now using the length and substituting the above into the length formula:

$$\sqrt{(x_B - 1)^2 + (3x_B - 3)^2} = \sqrt{10}$$

This gives $(x_B - 1)^2 = \pm 1$, so $x_B = 2$ or $x_B = 0$ and hence B has either coordinates $(2, 1)$ or $(0, -5)$.