

## Binomial theorem [50 marks]

1. [Maximum mark: 4]

24N.2.SL.TZ1.2

Find the coefficient of  $x^8$  in the expansion of  $(2x - 5)^{11}$ .

[4]

Markscheme

### EITHER

attempt to form a product of binomial coefficient, a power of  $2x$  and a power of  $-5$  seen (M1)

$${}^{11}C_8(2x)^8(-5)^3 \text{ OR } {}^{11}C_3(2x)^8(-5)^3 \text{ OR } 165 \times (2x)^8(-5)^3. \quad (A1)(A1)$$

**Note:** Award A1 for  ${}^{11}C_8$  or  ${}^{11}C_3$  or 165, A1 for  $(2x)^8(-5)^3$ .

### OR

attempt to use the general term (M1)

$${}^{11}C_r(2x)^{11-r}(-5)^r \text{ and } r = 3 \quad (A1)(A1)$$

### THEN

$$-5280000 \text{ (exact)} \quad A1$$

**Note:** Award A0 for a final answer of  $-5280000x^8$ .

[4 marks]

2. [Maximum mark: 6]

23N.1.SL.TZ1.6

The binomial expansion of  $(1 + kx)^n$  is given by

$$1 + \frac{9x}{2} + 15k^2x^2 + \dots + k^nx^n, \text{ where } n \in \mathbb{Z}^+ \text{ and } k \in \mathbb{Q}.$$

Find the value of  $n$  and the value of  $k$ .

[6]

Markscheme

attempt to apply binomial expansion (M1)

$$(1 + kx)^n = 1 + {}^n C_1 kx + {}^n C_2 k^2 x^2 + \dots \text{ OR } {}^n C_1 k = \frac{9}{2} \text{ OR } {}^n C_2 = 15$$

$$nk = \frac{9}{2} \quad (A1)$$

$$n \frac{{}^{(n-1)}}{2} = 15 \text{ OR } \frac{n!}{(n-2)!2!} = 15 \quad (A1)$$

$$(n^2 - n - 30 = 0) \text{ OR } n(n - 1) = 30$$

valid attempt to solve (M1)

$(n - 6)(n - 5) = 0$  OR  $6(6 - 1) = 30$  OR finding correct value in Pascal's triangle

$$\Rightarrow n = 6 \quad A1$$

$$\Rightarrow k = \frac{3}{4} \quad A1$$

**Note:** If candidate finds  $n = 6$  with no working shown, award **M1A0A0M1A1A0**.

If candidate finds  $n = 6$  and  $k = \frac{3}{4}$  with no working shown, award **M1A0A0M1A1A1**.

[6 marks]

3. [Maximum mark: 7]

23M.2.SL.TZ1.6

The coefficient of  $x^6$  in the expansion of  $(ax^3 + b)^8$  is 448.

The coefficient of  $x^6$  in the expansion of  $(ax^3 + b)^{10}$  is 2880.

Find the value of  $a$  and the value of  $b$ , where  $a, b > 0$ .

[7]

Markscheme

product of a binomial coefficient, a power of  $ax^3$  and a power of  $b$  seen (M1)

evidence of correct term chosen

for  $n = 8 : r = 2$  (or  $r = 6$ ) OR for  $n = 10 : r = 2$  (or  $r = 8$ ) (A1)

correct equations (may include powers of  $x$ ) A1A1

$${}_8C_2 a^2 b^6 = 448 \quad (28a^2 b^6 = 448 \Rightarrow a^2 b^6 = 16),$$

$${}_{10}C_2 a^2 b^8 = 2880 \quad (45a^2 b^8 = 2880 \Rightarrow a^2 b^8 = 64)$$

attempt to solve their system in  $a$  and  $b$  algebraically or graphically (M1)

$$b = 2; a = \frac{1}{2} \quad A1A1$$

**Note:** Award a maximum of (M1)(A1)A1A1(M1)A1A0 for  $b = \pm 2$  and/or  $a = \pm \frac{1}{2}$ .

[7 marks]

4. [Maximum mark: 6]

22N.2.SL.TZ0.6

Consider the expansion of  $\frac{(ax+1)^9}{21x^2}$ , where  $a \neq 0$ . The coefficient of the term in  $x^4$  is  $\frac{8}{7}a^5$ .

Find the value of  $a$ .

[6]

Markscheme

**Note:** Do not award any marks if there is clear evidence of adding instead of multiplying, for example  ${}^9C_r + (ax)^{9-r} + (1)^r$ .

valid approach for expansion (must be the product of a binomial coefficient with

$n = 9$  and a power of  $ax$  (M1)

$${}^9C_r(ax)^{9-r}(1)^r \text{ OR } {}^9C_{9-r}(ax)^r(1)^{9-r} \text{ OR} \\ {}^9C_0(ax)^0(1)^9 + {}^9C_1(ax)^1(1)^8 + \dots$$

recognizing that the term in  $x^6$  is needed (M1)

$$\frac{\text{Term in } x^6}{21x^2} = kx^4 \text{ OR } r = 6 \text{ OR } r = 3 \text{ OR } 9 - r = 6$$

correct term or coefficient in binomial expansion (seen anywhere) (A1)

$${}^9C_6(ax)^6(1)^3 \text{ OR } {}^9C_3a^6x^6 \text{ OR } 84(a^6x^6)(1) \text{ OR } 84a^6$$

**EITHER**

correct term in  $x^4$  or coefficient (may be seen in equation) (A1)

$$\frac{{}^9C_6}{21}a^6x^4 \text{ OR } 4a^6x^4 \text{ OR } 4a^6$$

Set their term in  $x^4$  or coefficient of  $x^4$  equal to  $\frac{8}{7}a^5x^4$  or  $\frac{8}{7}a^5$  (do not accept other powers of  $x$ ) (M1)

$$\frac{{}^9C_3}{21}a^6x^4 = \frac{8}{7}a^5x^4 \text{ OR } 4a^6 = \frac{8}{7}a^5$$

**OR**

correct term in  $x^6$  or coefficient of  $x^6$  (may be seen in equation) (A1)

$$84a^6x^6 \text{ OR } 84a^6$$

set their term in  $x^6$  or coefficient of  $x^6$  equal to  $24a^5x^6$  or  $24a^5$  (do not accept other powers of  $x$ ) (M1)

$$84a^6x^6 = 24a^5x^6 \text{ OR } 84a = 24$$

**THEN**

$$a = \frac{2}{7} \approx 0.286 (0.285714 \dots) \quad A1$$

**Note:** Award **A0** for the final mark for  $a = \frac{2}{7}$  and  $a = 0$ .

[6 marks]

5. [Maximum mark: 5]

22M.1.AHL.TZ1.6

Consider the expansion of  $\left(8x^3 - \frac{1}{2x}\right)^n$  where  $n \in \mathbb{Z}^+$ . Determine all possible values of  $n$  for which the expansion has a non-zero constant term.

[5]

Markscheme

**EITHER**

attempt to obtain the general term of the expansion

$$T_{r+1} = {}_n C_r (8x^3)^{n-r} \left(-\frac{1}{2x}\right)^r \text{ OR } T_{r+1} = {}_n C_{n-r} (8x^3)^r \left(-\frac{1}{2x}\right)^{n-r}$$

(M1)

**OR**

recognize power of  $x$  starts at  $3n$  and goes down by 4 each time (M1)

**THEN**

recognizing the constant term when the power of  $x$  is zero (or equivalent)

(M1)

$$r = \frac{3n}{4} \text{ or } n = \frac{4}{3}r \text{ or } 3n - 4r = 0 \text{ OR } 3r - (n - r) = 0 \text{ (or equivalent)} \quad A1$$

$r$  is a multiple of 3 ( $r = 3, 6, 9, \dots$ ) or one correct value of  $n$  (seen anywhere)

(A1)

$$n = 4k, k \in \mathbb{Z}^+ \quad A1$$

**Note:** Accept  $n$  is a (positive) multiple of 4 or  $n = 4, 8, 12, \dots$

Do not accept  $n = 4, 8, 12$

**Note:** Award full marks for a correct answer using trial and error approach showing  $n = 4, 8, 12, \dots$  and for recognizing that this pattern continues.

[5 marks]

6. [Maximum mark: 5]

21M.1.SL.TZ2.4

In the expansion of  $(x + k)^7$ , where  $k \in \mathbb{R}$ , the coefficient of the term in  $x^5$  is 63.

Find the possible values of  $k$ .

[5]

Markscheme

**EITHER**

attempt to use the binomial expansion of  $(x + k)^7$  (M1)

$${}^7C_0 x^7 k^0 + {}^7C_1 x^6 k^1 + {}^7C_2 x^5 k^2 + \dots \text{ (or)} \\ {}^7C_0 k^7 x^0 + {}^7C_1 k^5 x^1 + {}^7C_2 k^3 x^2 + \dots$$

identifying the correct term  ${}^7C_2 x^5 k^2$  (or  ${}^7C_5 k^2 x^5$ ) (A1)

**OR**

attempt to use the general term  ${}^7C_r x^r k^{7-r}$  (or  ${}^7C_r k^r x^{7-r}$ ) (M1)

$$r = 2 \text{ (or } r = 5) \quad (A1)$$

**THEN**

$${}^7C_2 = 21 \text{ (or } {}^7C_5 = 21 \text{ (seen anywhere))} \quad (A1)$$

$$21x^5k^2 = 63x^5 \quad (21k^2 = 63, k^2 = 3) \quad A1$$

$$k = \pm\sqrt{3} \quad A1$$

**Note:** If working shown, award *M1A1A1A1A0* for  $k = \sqrt{3}$ .

[5 marks]

7. [Maximum mark: 5]

21M.2.SL.TZ1.6

Consider the expansion of  $(3 + x^2)^{n+1}$ , where  $n \in \mathbb{Z}^+$ .

Given that the coefficient of  $x^4$  is 20412, find the value of  $n$ .

[5]

Markscheme

**METHOD 1**

product of a binomial coefficient, a power of 3 (and a power of  $x^2$ ) seen (M1)

evidence of correct term chosen (A1)

$${}^{n+1}C_2 \times 3^{n+1-2} \times (x^2)^2 \left( = \frac{n(n+1)}{2} \times 3^{n-1} \times x^4 \right) \text{ OR } n - r = 1$$

equating their coefficient to 20412 or their term to  $20412x^4$  (M1)

**EITHER**

$${}^{n+1}C_2 \times 3^{n-1} = 20412 \quad (A1)$$

**OR**

$${}^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6 \quad (A1)$$

**THEN**

$$n = 7 \quad A1$$

**METHOD 2**

$$3^{n+1} \left(1 + \frac{x^2}{3}\right)^{n+1}$$

product of a binomial coefficient, and a power of  $\frac{x^2}{3}$  **OR**  $\frac{1}{3}$  seen *(M1)*

evidence of correct term chosen *(A1)*

$$3^{n+1} \times \frac{n(n+1)}{2!} \times \left(\frac{x^2}{3}\right)^2 \left(= \frac{3^{n-1}}{2} n(n+1)x^4\right)$$

equating their coefficient to 20412 or their term to  $20412x^4$  *(M1)*

$$3^{n-1} \times \frac{n(n+1)}{2} = 20412 \quad (A1)$$

$$n = 7 \quad A1$$

*[5 marks]*

8. [Maximum mark: 6]

20N.2.SL.TZ0.S\_5

Consider the expansion of  $\left(3x^2 - \frac{k}{x}\right)^9$ , where  $k > 0$ .

The coefficient of the term in  $x^6$  is 6048. Find the value of  $k$ .

[6]

Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for  $r$ ). (M1)

eg

$$\binom{9}{r} (3x^2)^{9-r} \left(-\frac{k}{x}\right)^r, (3x^2)^9 + \binom{9}{1} (3x^2)^8 \left(-\frac{k}{x}\right)^1 + \binom{9}{2} (3x^2)^7 \left(-\frac{k}{x}\right)^2 + \dots$$

valid attempt to identify correct term (M1)

$$\text{eg } 2(9-r) - r = 6, (x^2)^r (x^{-1})^{9-r} = x^6$$

identifying correct term (may be indicated in expansion) (A1)

$$\text{eg } r = 4, r = 5$$

correct term or coefficient in binominal expansion (A1)

$$\text{eg } \binom{9}{4} (3x^2)^5 \left(-\frac{k}{x}\right)^4, 126(243x^{10}) \left(\frac{k^4}{x^4}\right), 30618k^4$$

correct equation in  $k$  (A1)

$$\text{eg } \binom{9}{4} (243)(k^4)x^6 = 6048x^6, 30618k^4 = 6048$$

$$k = \frac{2}{3} \text{ (exact) } 0.667 \quad \text{A1 N3}$$

**Note:** Do not award A1 if additional answers given.

[6 marks]

Find the term independent of  $x$  in the expansion of  $\frac{1}{x^3} \left( \frac{1}{3x^2} - \frac{x}{2} \right)^9$ .

[6]

Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of Binomial expansion to find a term in either

$$\left( \frac{1}{3x^2} - \frac{x}{2} \right)^9, \left( \frac{1}{3x^{7/3}} - \frac{x^{2/3}}{2} \right)^9, \left( \frac{1}{3} - \frac{x^3}{2} \right)^9, \left( \frac{1}{3x^3} - \frac{1}{2} \right)^9 \text{ or} \\ (2 - 3x^3)^9 \quad (M1)(A1)$$

**Note:** Award **M1** for a product of three terms including a binomial coefficient and powers of the two terms, and **A1** for a correct expression of a term in the expansion.

finding the powers required to be 2 and 7  $(M1)(A1)$

$$\text{constant term is } {}_9C_2 \times \left( \frac{1}{3} \right)^2 \times \left( -\frac{1}{2} \right)^7 \quad (M1)$$

**Note:** Ignore all  $x$ 's in student's expression.

$$\text{therefore term independent of } x \text{ is } -\frac{1}{32} (= -0.03125) \quad A1$$

[6 marks]