Extra revision [88 marks]

1. [Maximum mark: 13] SPM.2.SL.TZ0.9

Consider a function f, such that $f(x) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

(a) Find the period of f. [2]

Markscheme

correct approach ***A1***

eg $\frac{\pi}{6} = \frac{2\pi}{period}$ (or equivalent)

period = 12 ***A1***

[2 marks]

The function f has a local maximum at the point (2, 21.8) , and a local minimum at (8, 10.2).

(b.i) Find the value of b. [2]

Markscheme

valid approach ***(M1)***

eg $\frac{\max+\min}{2}$ $b = \max - \text{amplitude}$

$\frac{21.8+10.2}{2}$, or equivalent

b = 16 ***A1***

[2 marks]

(b.ii) Hence, find the value of $f(6)$. [2]

Markscheme

attempt to substitute into **their** function ***(M1)***

$5.8\sin\left(\frac{\pi}{6}(6+1)\right)+16$

$f(6)=13.1$ ***A1***

[2 marks]

A second function g is given by $g(x)=p\sin\left(\frac{2\pi}{9}(x-3.75)\right)+q$, $0 \le x \le 10$; $p, q \in \mathbb{R}$.

The function g passes through the points (3, 2.5) and (6, 15.1).

(c) Find the value of p and the value of q. [5]

Markscheme

valid attempt to set up a system of equations ***(M1)***

two correct equations ***A1***

$p\sin\left(\frac{2\pi}{9}(3-3.75)\right)+q=2.5$,
$p\sin\left(\frac{2\pi}{9}(6-3.75)\right)+q=15.1$

valid attempt to solve system ***(M1)***

$p = 8.4; q = 6.7$ ***A1A1***

[5 marks]

(d) Find the value of x for which the functions have the greatest difference. [2]

Markscheme

attempt to use $|f(x) - g(x)|$ to find maximum difference ***(M1)***

$x = 1.64$ ***A1***

[2 marks]

2. [Maximum mark: 16] SPM.2.SL.TZ0.7

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

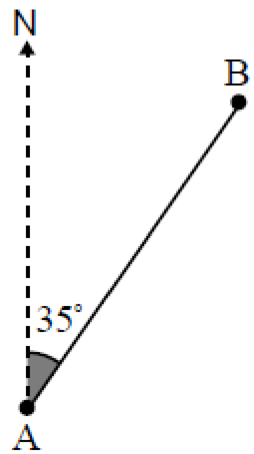
(a) Find the distance from point A to point B. [2]

Markscheme

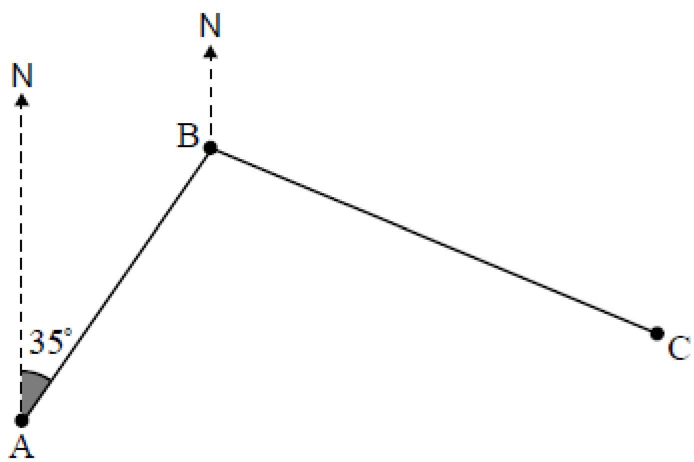
$\frac{4.2}{60} \times 45$ ***A1***

AB = 3.15 (km) ***A1***

[2 marks]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



(b.i) Show that $\hat{ABC}$ is 101°. [2]

Markscheme

66° or (180 – 114) ***A1***

35 + 66 ***A1***

$\hat{ABC} = 101°$ ***AG***

[2 marks]

(b.ii) Find the distance from the camp to point C. [3]

Markscheme

attempt to use cosine rule ***(M1)***

$AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101°$ (or equivalent) ***A1***

AC = 6.05 (km) ***A1***

[3 marks]

(c) Find $\hat{BCA}$. [3]

Markscheme

valid approach to find angle BCA ***(M1)***

eg sine rule

correct substitution into sine rule ***A1***

eg $\frac{\sin\left(\hat{BCA}\right)}{3.15} = \frac{\sin 101}{6.0507\ldots}$

$\hat{BCA} = 30.7°$ ***A1***

[3 marks]

Adam’s friend Jacob wants to hike directly from the camp to meet Adam at point C.

(d) Find the bearing that Jacob must take to point C. [3]

Markscheme

$\mathrm{B\hat{A}C}$ = 48.267 (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

eg 48.267 + 35

bearing = 83.3° (accept 083°) ***A1***

[3 marks]

(e) Jacob hikes at an average speed of 3.9 km/h.

Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Markscheme

attempt to use $\mathrm{time} = \frac{\mathrm{distance}}{\mathrm{speed}}$ ***M1***

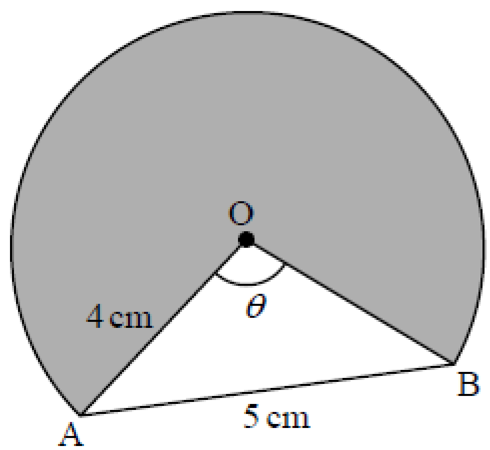
$\frac{6.0507}{3.9}$ or 0.065768 km/min ***(A1)***

t = 93 (minutes) ***A1***

[3 marks]

3. [Maximum mark: 6] SPM.2.SL.TZ0.2

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and $A\hat{O}B = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

Markscheme

METHOD 1

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2+4^2-5^2}{2\times4\times4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

METHOD 2

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

(b) Find the area of the shaded region. [3]

Markscheme

attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35\ldots)$ ***A1***

= 39.5 (cm^2) ***A1***

[3 marks]

4. [Maximum mark: 6] SPM.2.SL.TZ0.3

On 1st January 2020, Laurie invests $*P* in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, *r*.

(a) Find the value of *r*, giving your answer to four significant figures. [3]

Markscheme

$\left(1 + \frac{5.5}{4 \times 100}\right)^4$ ***(M1)(A1)***

1.056 ***A1***

[3 marks]

(b) Laurie makes no further deposits to or withdrawals from the account.

Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

Markscheme

EITHER

$2P = P \times \left(1 + \frac{5.5}{100 \times 4}\right)^{4n}$ **OR** $2P = P \times (\text{their }(a))^{m}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

PV = ±1
FV = ∓1
I% = 5.5
P/Y = 4
C/Y = 4
n = 50.756… ***(M1)(A1)***

OR

PV = ±1
FV = ∓2
I% = 100(their (a) − 1)
P/Y = 1
C/Y = 1 ***(M1)(A1)***

THEN

⇒ 12.7 years

Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

5. [Maximum mark: 6] EXN.1.SL.TZ0.5

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a,\ b \in \mathbb{R}$.

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b. [6]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$(f \circ g)(x) = ax + b - 2$ **(M1)**

$(f \circ g)(2) = -3 \Rightarrow 2a + b - 2 = -3\ (2a + b = -1)$ **A1**

$(g \circ f)(x) = a(x - 2) + b$ **(M1)**

$(g \circ f)(1) = 5 \Rightarrow -a + b = 5$ **A1**

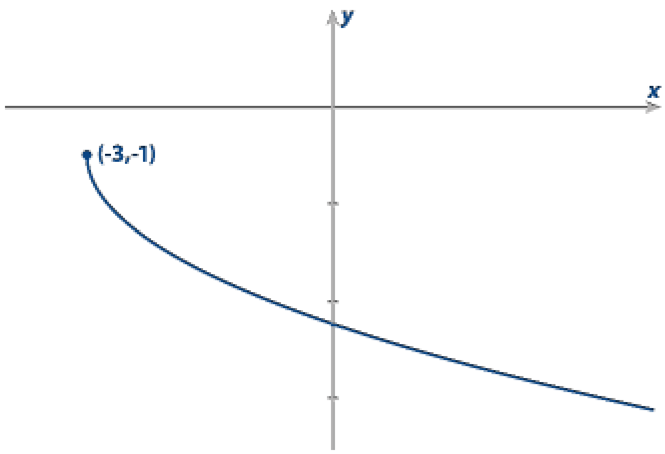
a valid attempt to solve their two linear equations for a and b **M1**

so $a = -2$ and $b = 3$ **A1**

[6 marks]

6. [Maximum mark: 14] EXN.1.SL.TZ0.8

The following diagram shows the graph of $y = -1 - \sqrt{x + 3}$ for $x \geq -3$.



(a) Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. [3]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

for example,

a reflection in the x-axis (in the line $y = 0$) **A1**

a horizontal translation (shift) 3 units to the left **A1**

a vertical translation (shift) down by 1 unit **A1**

Note: Award **A1** for each correct transformation applied in a correct position in the sequence. Do not accept use of the "move" for a translation.

Note: Award **A1A1A1** for a correct alternative sequence of transformations. For example, a vertical translation (shift) down by 1 unit, followed by a horizontal translation (shift) 3 units to the left and then a reflection in the line $y = -1$.

[3 marks]

A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

(b) State the range of f. [1]

Markscheme

range is $f(x) \leq -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$, $(-\infty, -1]$ or $y \leq -1$.

[1 mark]

(c) Find an expression for $f^{-1}(x)$, stating its domain. [5]

Markscheme

$-1 - \sqrt{y+3} = x$ **M1**

Note: Award **M1** for interchanging x and y (can be done at a later stage).

$\sqrt{y+3} = -x - 1 (= -(x+1))$ **A1**

$y + 3 = (x+1)^2$ **A1**

so $f^{-1}(x) = (x+1)^2 - 3 \ (f^{-1}(x) = x^2 + 2x - 2)$ **A1**

domain is $x \le -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$ or $(-\infty, -1]$.

[5 marks]

(d) Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. [5]

Markscheme

the point of intersection lies on the line $y = x$

EITHER

$(x+1)^2 - 3 = x$ **M1**

attempts to solve their quadratic equation **M1**

for example, $(x+2)(x-1) = 0$ or $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$ $\left(x = \frac{-1 \pm 3}{2}\right)$

OR

$-1 - \sqrt{x+3} = x$ **M1**

$$\left(-1-\sqrt{x+3}\right)^2 = x^2 \Rightarrow 2\sqrt{x+3}+x+4 = x^2$$

substitutes $2\sqrt{x+3} = -2(x+1)$ to obtain
$-2(x+1)+x+4 = x^2$

attempts to solve their quadratic equation **M1**

for example, $(x+2)(x-1) = 0$ or
$x = \frac{-1\pm\sqrt{1^2-4(1)(-2)}}{2}$ $\left(x = \frac{-1\pm3}{2}\right)$

THEN

$x = -2, 1$ **A1**

as $x \leq -1$, the only solution is $x = -2$ **R1**

so the coordinates of the point of intersection are $(-2, \ -2)$ **A1**

Note: Award **R0A1** if $(-2, \ -2)$ is stated without a valid reason given for rejecting $(1, \ 1)$.

[5 marks]

7. [Maximum mark: 5] EXN.2.SL.TZ0.5

The quadratic equation $(k-1)x^2 + 2x + (2k-3) = 0$, where $k \in \mathbb{R}$, has real distinct roots.

Find the range of possible values for k. [5]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

attempts to find an expression for the discriminant, Δ, in terms of k **(M1)**

$\Delta = 4 - 4(k-1)(2k-3) \ \left(= -8k^2 + 20k - 8\right)$ **(A1)**

Note: Award **M1A1** for finding $x = \frac{-2 \pm \sqrt{4-4(k-1)(2k-3)}}{2(k-1)}$.

attempts to solve $\Delta > 0$ for k **(M1)**

Note: Award **M1** for attempting to solve $\Delta = 0$ for k.

$\frac{1}{2} < k < 2$ **A1A1**

Note: Award **A1** for obtaining critical values $k = \frac{1}{2}, 2$ and **A1** for correct inequality signs.

[5 marks]

8. [Maximum mark: 12] EXN.2.SL.TZ0.7

Helen and Jane both commence new jobs each starting on an annual salary of $\$70,000$. At the start of each new year, Helen receives an annual salary increase of $\$2400$.

Let $\$H_n$ represent Helen's annual salary at the start of her nth year of employment.

(a) Show that $H_n = 2400n + 67\,600$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

uses $H_n = H_1 + (n-1)d$ with $H_1 = 70\,000$ and $d = 2400$ **(M1)**

$H_n = 70\,000 + 2400(n-1)$ **A1**

so $H_n = 2400n + 67\,600$ **AG**

[2 marks]

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

Jane's annual salary, $\$J_n$, at the start of her nth year of employment is given by $J_n = 70\,000(1.03)^{n-1}$.

(b) Given that J_n follows a geometric sequence, state the value of the common ratio, r. [1]

Markscheme

$r = 1.03$ **A1**

[1 mark]

At the start of year N, Jane's annual salary exceeds Helen's annual salary for the first time.

(c.i) Find the value of N. [3]

Markscheme

evidence of use of an appropriate table or graph or GDC numerical solve feature to find the value of N such that $J_n > H_n$ **(M1)**

EITHER

for example, an excerpt from an appropriate table

N	H_n	J_n
11	94 000	94 074

(A1)

OR

for example, use of a GDC numerical solve feature to obtain $N = 10.800\ldots$ **(A1)**

Note: Award **A1** for an appropriate graph. Condone use of a continuous graph.

THEN

$N = 11$ **A1**

[3 marks]

(c.ii) For the value of N found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar. [2]

Markscheme

$H_{11} = 94\,000\,(\$)$ **A1**

$J_{11} = 94\,074\,(\$)$ **A1**

Helen's annual salary is $\$94\,000$ and Jane's annual salary is $\$94\,074$

Note: Award **A1** for a correct H_{11} value and **A1** for a correct J_{11} value seen in part (c) (i).

[2 marks]

(d) Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar. [4]

Markscheme

at the start of the 10th year, Jane will have worked for 9 years so the value of S_9 is required **R1**

Note: Award **R1** if S_9 is seen anywhere.

uses $S_n = \frac{J_1(r^n - 1)}{r - 1}$ with $J_1 = 70\,000$, $r = 1.03$ and $n = 9$ **(M1)**

Note: Award **M1** if $n = 10$ is used.

$$S_9 = \frac{70\,000\left((1.03)^9 - 1\right)}{1.03 - 1} = 711\,137.42\ldots \quad \textbf{(A1)}$$

$$= 711\,137\,(\$)$$

Jane's total earnings are $\$711\,137$ (correct to the nearest dollar)

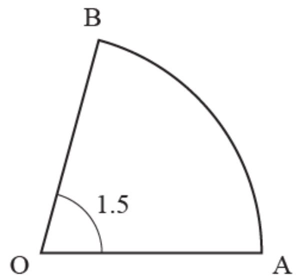
[4 marks]

9. [Maximum mark: 5] 24N.1.SL.TZ2.2

Points A and B lie on a circle with centre O and radius r cm, where $\mathrm{A\hat{O}B} = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $48\ \mathrm{cm}^2$.

(a) Find the value of r. [3]

Markscheme

$\frac{1}{2}r^2\theta = 48$ OR $\frac{1}{2}r^2(1.5) = 48$ ***(A1)***

attempt to solve their equation to find r or r^2 ***(M1)***

Note: To award the ***M1***, candidate's equation must include r^2 and $\theta = 1.5$, and they must attempt to isolate r^2 or r.

$r^2 = 64$

$r = 8$ (cm) ***A1***

[3 marks]

(b) Hence, find the perimeter of sector OAB. [2]

Markscheme

evidence of summing the two radii and the arc length ***(M1)***

perimeter $= 2r + r\theta$

$= 16 + 8(1.5)$

$= 28$ (cm) ***A1***

[2 marks]

10. [Maximum mark: 5] 24N.1.AHL.TZ0.6

(a) Solve $2x^2 - 15x + 18 < 0$. [3]

Markscheme

attempt to find critical values ***(M1)***

$x = \frac{3}{2},\ x = 6$ ***(A1)***

$\frac{3}{2} < x < 6$ ***A1***

Note: Allow equivalent, and/or interval notation.

[3 marks]

(b) The function f is defined by $f(x) = \sqrt{2x^2 - 15x + 18}$, where $x \in \mathbb{R}, x \leq k$.

Find the greatest value of k for which f^{-1} exists, justifying your answer. [2]

Markscheme

$k = \frac{3}{2}$ ***A1***

since we require $2x^2 - 15x + 18 \geq 0$ (and f must be one to one)
R1

OR

Does not obey horizontal line test for $x \geq \frac{3}{2}$ ***R1***

[2 marks]