Extra revision part 2 [39 marks]

1. [Maximum mark: 5] 24N.2.AHL.TZ0.5

Consider the function $h(x) = \log_{10}\left(4x^2 - rx + r - 1\right)$, where $x \in \mathbb{R}$.

Find the possible values of r. [5]

Markscheme

METHOD 1

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

(discriminant $=$) $(-r)^2 - 4(4)(r-1) \quad \left(= r^2 - 16r + 16\right)$ (seen anywhere) ***(A1)***

$1.07179\ldots \quad \left(= 8 - 4\sqrt{3}\right)$ AND
$14.9282\ldots \quad \left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

recognition that discriminant of $4x^2 - rx + r - 1$ is less than zero ***(M1)***

$1.07 < r < 14.9 \quad \left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

METHOD 2

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) ***R1***

EITHER

minimum when

$x = \frac{r}{8} \Rightarrow \left(y =\right) 4\left(\frac{r}{8}\right)^2 - r\left(\frac{r}{8}\right) + r - 1 \ \left(> 0\right)$ ***(A1)***

attempt to solve their inequality for y (must be in terms of r and r^2) ***(M1)***

OR

$x < 1 \Rightarrow r > \frac{4x^2-1}{x-1}$ OR $x > 1 \Rightarrow r < \frac{4x^2-1}{x-1}$ ***(A1)***

attempt to find local minumum AND local maximum of $r = \frac{4x^2-1}{x-1}$ ***(M1)***

THEN

$\left(r >\right) 1.07179\ldots \quad \left(= 8 - 4\sqrt{3}\right)$ AND
$\left(r <\right) 14.9282\ldots \quad \left(= 8 + 4\sqrt{3}\right)$ (seen anywhere) ***(A1)***

$1.07 < r < 14.9 \quad \left(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3}\right)$ ***A1***

Note: Accept $1.08 \le r \le 14.9$.

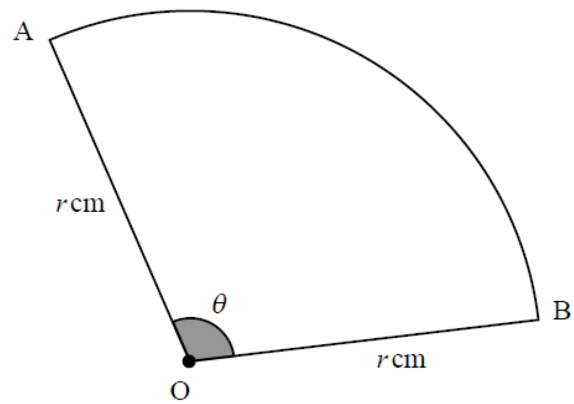
[5 marks]

2. [Maximum mark: 8] 24M.1.SL.TZ1.4

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\ \text{cm}$ and the area of the sector is $6.25\ \text{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

Markscheme

$2r + r\theta = 10$ ***A1***

$\frac{1}{2}r^2\theta = 6.25$ ***A1***

attempt to eliminate θ to obtain an equation in r ***M1***

correct intermediate equation in r ***A1***

$10 - 2r = \frac{25}{2r}$ OR $\frac{10}{r} - 2 = \frac{25}{2r^2}$ OR $\frac{1}{2}r^2\left(\frac{10}{r} - 2\right) = 6.25$
OR $1.25 + 2r^2 = 10r$

$4r^2 - 20r + 25 = 0$ ***AG***

[4 marks]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

Markscheme

attempt to solve quadratic by factorizing or use of formula or completing the square ***(M1)***

$(2r-5)^2 = 0$ OR $r = \frac{20 \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} \left(= \frac{20 \pm \sqrt{400-400}}{8}\right)$

$r = \frac{5}{2}$ ***A1***

attempt to substitute their value of r into their perimeter or area equation ***(M1)***

$\theta = \frac{10-2\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)}$ or $\theta = \frac{25}{2\left(\frac{5}{2}\right)^2}$

$\theta = 2$ ***A1***

[4 marks]

3. [Maximum mark: 5] 24M.1.SL.TZ1.6

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

(a) Find an expression for S_n in the form $\frac{a^n-1}{b}$, where $a,\ b \in \mathbb{Z}^+$. [1]

Markscheme

$S_n = \frac{10^n-1}{9}$ ***A1***

$(a = 10, b = 9)$

[1 mark]

(b) Hence, show that $S_1 + S_2 + S_3 + \ldots + S_n = \frac{10(10^n-1)-9n}{81}$. [4]

Markscheme

METHOD 1

$$S_1 + S_2 + S_3 + \ldots + S_n$$

$$= \frac{10-1}{9} + \frac{10^2-1}{9} + \ldots + \frac{10^n-1}{9} \qquad \textit{(A1)}$$

$$= \frac{10-1+10^2-1+10^3-1+\ldots+10^n-1}{9} \text{ OR } \frac{9(10-1+10^2-1+10^3-1+\ldots+10^n-1)}{81}$$

attempt to use geometric series formula on powers of 10, and collect -1's together ***M1***

$10 + 10^2 + 10^3 + \ldots + 10^n = \frac{10(10^n-1)}{10-1}$ and $-1 - 1 - 1 \ldots = -n$ ***A1***

$$= \frac{\frac{10(10^n-1)}{10-1} - n}{9} \text{ OR } \frac{9\left(\frac{10(10^n-1)}{10-1}\right) - 9n}{81} \qquad \textit{A1}$$

Note: Award ***A1*** for any correct intermediate expression.

$$= \frac{10(10^n-1)-9n}{81} \qquad \textit{AG}$$

METHOD 2

attempt to create sum using sigma notation with S_n ***M1***

$$\sum_{i=1}^{n} \frac{10^i-1}{9} \quad \left(= \frac{1}{9}\left(\sum_{i=1}^{n} 10^i - \sum_{i=1}^{n} 1\right)\right)$$

$$\sum_{i=1}^{n} 10^i = \frac{10(10^n-1)}{9}$$ ***A1***

$$\sum_{i=1}^{n} 1 = n$$ ***A1***

$$= \frac{1}{9}\left(\frac{10(10^n-1)}{9} - n\right) \text{ OR } \frac{1}{9}\left(\frac{10(10^n-1)-9n}{9}\right)$$ ***A1***

$$= \frac{10(10^n-1)-9n}{81}$$ ***AG***

[4 marks]

4. [Maximum mark: 4] 24M.1.SL.TZ2.2

Solve $\tan(2x - 5^\circ) = 1$ for $0^\circ \le x \le 180^\circ$. [4]

Markscheme

$\tan^{-1} 1 = 45^\circ$ or equivalent ***(A1)***

attempt to equate $2x - 5^\circ$ to their reference angle ***(M1)***

Note: Do not accept $2x - 5^\circ = 1$.

$2x - 5^\circ = 45^\circ, (225^\circ)$

$x = 25^\circ, 115^\circ$ ***A1A1***

Note: Do not award the final ***A1*** if any additional solutions are seen.

[4 marks]

5. [Maximum mark: 5] 24M.1.SL.TZ2.3

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

Markscheme

valid attempt to solve a quadratic equation (factorising, use of formula, completing square, or otherwise) ***(M1)***

$(3m - 1)(m + 2) = 0$ OR $m = \frac{-5 \pm \sqrt{25+24}}{6}$ (or equivalent) ***(A1)***

$m = \frac{1}{3}$, $m = -2$ ***A1***

[3 marks]

(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

Markscheme

setting their m-value(s) $= 3^x$ OR recognising a quadratic in 3^x ***(M1)***

$3^x = \frac{1}{3}$ (or $3^x = -2$) OR $3 \times (3^x)^2 + 5 \times 3^x - 2 = 0$

$x = -1$ ***A1***

Note: Award the final ***A1*** if candidate's answer includes $x = -1$ and $x = \log_3 (-2)$. Award ***A0*** if other incorrect answers are given.

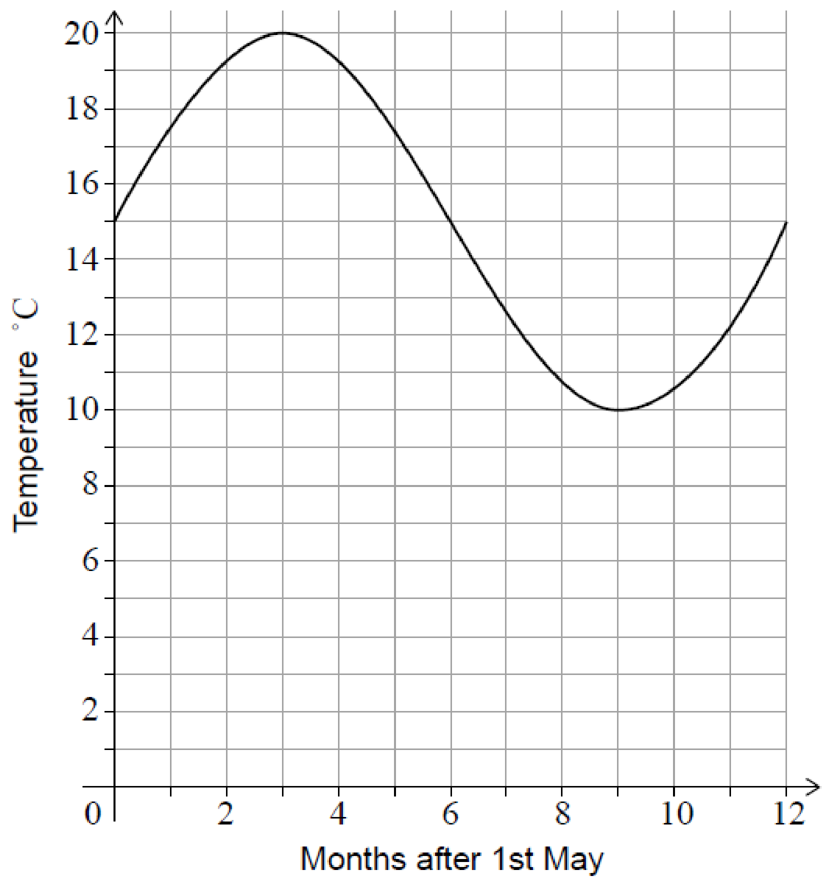
[2 marks]

6. [Maximum mark: 12] 24M.1.SL.TZ2.7

Alex only swims in the sea if the water temperature is at least $15^\circ\mathrm{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a\ \sin\ bx + c$ for $0 \leq x \leq 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(a) Show that $b = \frac{\pi}{6}$. [1]

Markscheme

$12b = 2\pi$ OR $(b =)\frac{2\pi}{12}$ OR $12 = \frac{2\pi}{b}$ *A1*

$b = \frac{\pi}{6}$ ***AG***

[1 mark]

(b) Write down the value of

(b.i) a; [1]

Markscheme

$a = 5$ ***A1***

[1 mark]

(b.ii) c. [1]

Markscheme

$c = 15$ ***A1***

[1 mark]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5\ \sin \frac{\pi}{6}x + 11$, where $0 \le x \le 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(c.i) find the midday water temperature on 1st October, five months after 1st May. [3]

Markscheme

attempt to substitute $x = 5$ into $g(x)$ ***(M1)***

$g(5) = 3.5 \sin \frac{5\pi}{6} + 11$

$\sin \frac{5\pi}{6} = \frac{1}{2}$ ***(A1)***

$g(5) = 3.5 \times \frac{1}{2} + 11$

$g(5) = 12.75 \left(= \frac{51}{4}\right)$ ***A1***

[3 marks]

(c.ii) show that the midday water temperature is never warm enough for Alex to swim. [3]

Markscheme

METHOD 1 (finding maximum temperature)

considering the maximum value of $\sin \frac{\pi}{6} x (= 1)$ OR $g\prime(x) = 0$ at maximum

OR maximum = vertical shift + amplitude (may be seen on a graph) ***(M1)***

$g_{\text{max}} = 3.5 + 11$ OR $\frac{\pi}{6} \cdot 3.5 \cos \left(\frac{\pi}{6} x\right) = 0$ OR $x = 3$

$g_{\text{max}} = 14.5$ ***A1***

$14.5 < 15$ (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

Note: Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***).

Worded conclusions are acceptable for the ***R1***, as long as the reasoning is clear that the water does not reach 15°, so not warm enough for Alex.

METHOD 2 (working with inequality)

$3.5\sin\left(\frac{\pi}{6}x\right)+11\geq 15$ ***(M1)***

$\sin\left(\frac{\pi}{6}x\right)\geq\frac{8}{7}$ ***A1***

sine values can never be greater than 1 (hence the midday water temperature is never warm enough for Alex to swim) ***R1***

Note: Do not award the R mark unless the previous marks been awarded (Do not award ***M1A0R1*** or ***M0A0R1***).

If candidate works with an equation throughout, the ***M1*** and ***A1*** may be awarded, if appropriate. A correct inequality is required for the ***R1*** to be awarded.

[3 marks]

(d) Alex compares the two models and finds that $g(x)=0.7f(x)+q$. Determine the value of q. [3]

Markscheme

EITHER

attempt to find $0.7f(x)$ OR $0.7f(x)+q$ ***(M1)***

$0.7f(x)=3.5\sin\frac{\pi}{6}x+10.5$ OR

$0.7f(x)+q=3.5\sin\frac{\pi}{6}x+10.5+q$ OR $10.5+q=11$ ***(A1)***

OR

attempt to find $0.7f(x)$ for a particular value of x ***(M1)***

eg maximum $20 \times 0.7 = 14$ ***(A1)***

THEN

$q = 0.5$ ***A1***

[3 marks]