Extra revision part 2 [39 marks]

1. [Maximum mark: 5] 24N.2.AHL.TZ0.5

Consider the function $h(x) = \log_{10}\left(4x^2 - rx + r - 1\right)$, where $x \in \mathbb{R}$.

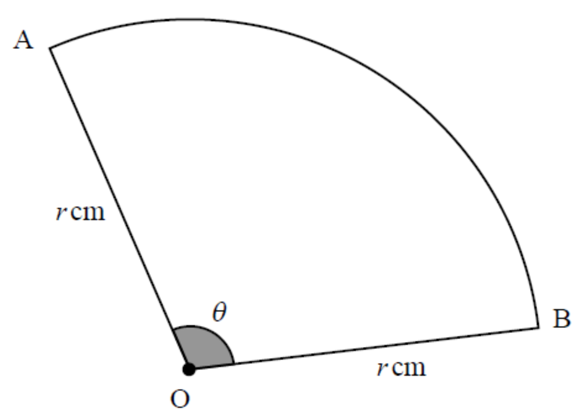
Find the possible values of r. [5]

2. [Maximum mark: 8] 24M.1.SL.TZ1.4

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is 10 cm and the area of the sector is $6.25\ \mathrm{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

3. [Maximum mark: 5] 24M.1.SL.TZ1.6

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Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

(a) Find an expression for S_n in the form $\frac{a^n-1}{b}$, where $a,\ b \in \mathbb{Z}^+$. [1]

(b) Hence, show that

$$S_1 + S_2 + S_3 + \ldots + S_n = \frac{10(10^n-1)-9n}{81}.$$ [4]

4. [Maximum mark: 4] 24M.1.SL.TZ2.2

Solve $\tan\,(2x - 5^\circ) = 1$ for $0^\circ \le x \le 180^\circ$. [4]

5. [Maximum mark: 5] 24M.1.SL.TZ2.3

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

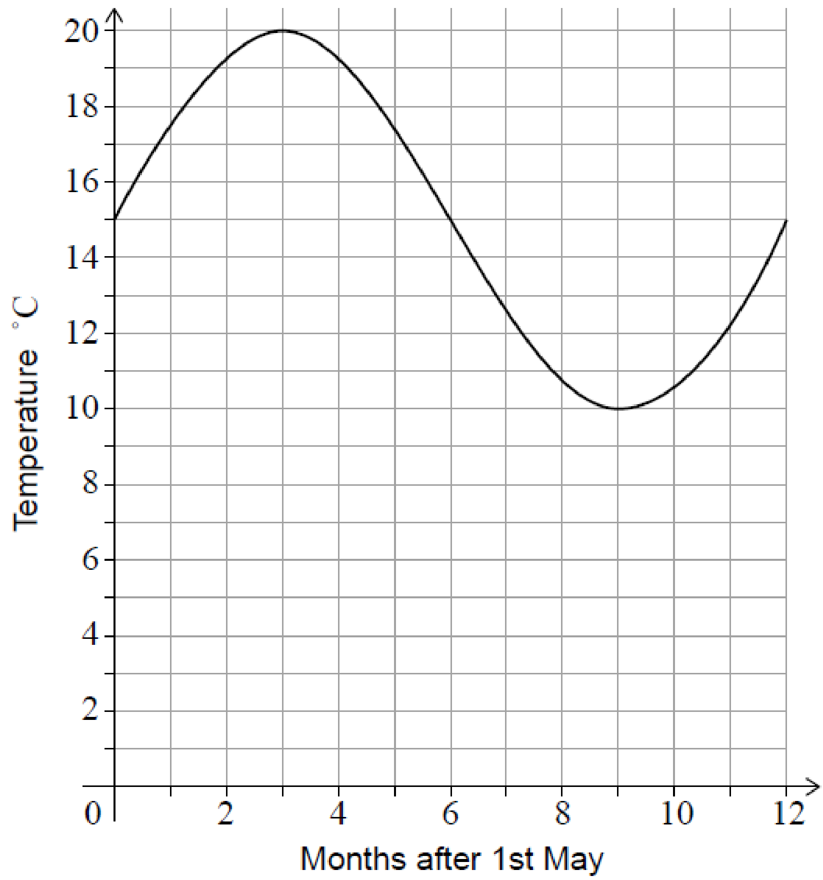
(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

6. [Maximum mark: 12] 24M.1.SL.TZ2.7

Alex only swims in the sea if the water temperature is at least $15\,^\circ\mathrm{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a\sin bx + c$ for $0 \le x \le 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Write down the value of

(b.i) a; [1]

(b.ii) c. [1]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5 \sin \frac{\pi}{6} x + 11$, where $0 \leq x \leq 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(c.i) find the midday water temperature on 1st October, five months after 1st May. [3]

(c.ii) show that the midday water temperature is never warm enough for Alex to swim. [3]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]