Extra revision [88 marks]

1. [Maximum mark: 13] SPM.2.SL.TZ0.9

Consider a function f, such that $f\left(x\right) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

(a) Find the period of f. [2]

The function f has a local maximum at the point (2, 21.8) , and a local minimum at (8, 10.2).

(b.i) Find the value of b. [2]

(b.ii) Hence, find the value of $f(6)$. [2]

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x-3.75)\right) + q$, $0 \le x \le 10$; $p, q \in \mathbb{R}$.

The function g passes through the points (3, 2.5) and (6, 15.1).

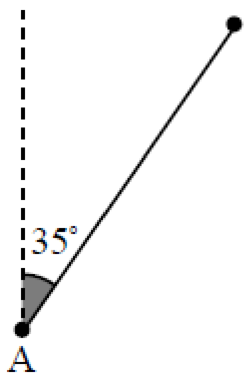
(c) Find the value of p and the value of q. [5]

(d) Find the value of x for which the functions have the greatest difference. [2]

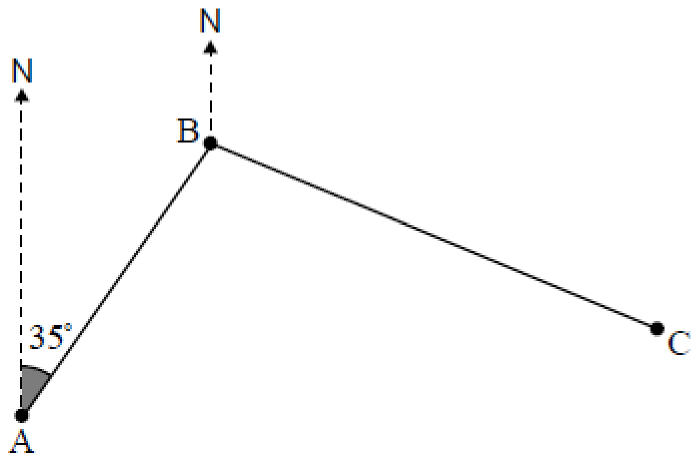
2. [Maximum mark: 16] SPM.2.SL.TZ0.7

Adam sets out for a hike from his camp at point A. He hikes at an average speed of 4.2 km/h for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a) Find the distance from point A to point B. [2]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of 4.6 km until he reaches point C.



(b.i) Show that $\hat{ABC}$ is 101°. [2]

(b.ii) Find the distance from the camp to point C. [3]

(c) Find $\hat{BCA}$. [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C .

(d) Find the bearing that Jacob must take to point C. [3]

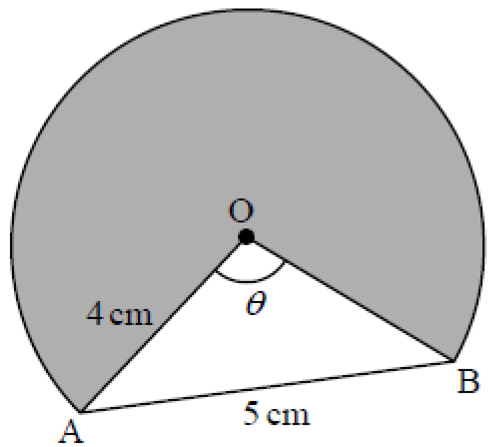
(e) Jacob hikes at an average speed of 3.9 km/h.

Find, to the nearest minute, the time it takes for Jacob to reach point C.

[3]

3. [Maximum mark: 6] SPM.2.SL.TZ0.2

The following diagram shows part of a circle with centre O and radius 4 cm.



Chord AB has a length of 5 cm and $A\hat{O}B = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

(b) Find the area of the shaded region. [3]

4. [Maximum mark: 6] SPM.2.SL.TZ0.3

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

(b) Laurie makes no further deposits to or withdrawals from the account.

Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

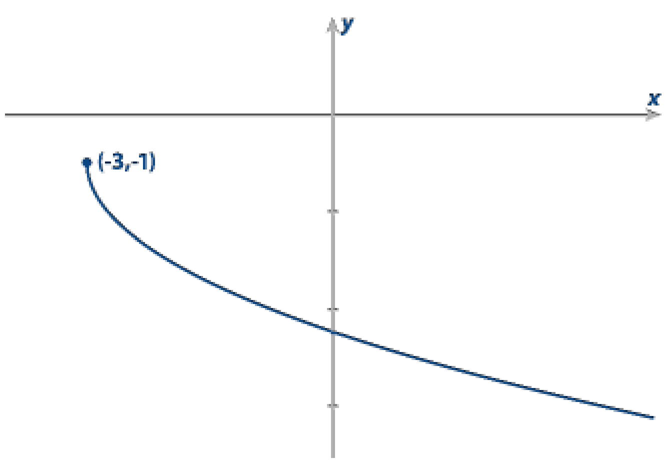
5. [Maximum mark: 6] EXN.1.SL.TZ0.5

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a,\ b \in \mathbb{R}$.

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b. [6]

6. [Maximum mark: 14] EXN.1.SL.TZ0.8

The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



(a) Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. [3]

A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

(b) State the range of f. [1]

(c) Find an expression for $f^{-1}(x)$, stating its domain. [5]

(d) Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. [5]

7. [Maximum mark: 5] EXN.2.SL.TZ0.5

The quadratic equation $(k-1)x^2 + 2x + (2k-3) = 0$, where $k \in \mathbb{R}$, has real distinct roots.

Find the range of possible values for k. [5]

8. [Maximum mark: 12] EXN.2.SL.TZ0.7

Helen and Jane both commence new jobs each starting on an annual salary of $\$70,000$. At the start of each new year, Helen receives an annual salary increase of $\$2400$.

Let $\$H_n$ represent Helen's annual salary at the start of her nth year of employment.

(a) Show that $H_n = 2400n + 67\,600$. [2]

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

Jane's annual salary, $\$J_n$, at the start of her nth year of employment is given by $J_n = 70\,000(1.03)^{n-1}$.

(b) Given that J_n follows a geometric sequence, state the value of the common ratio, r. [1]

At the start of year N, Jane's annual salary exceeds Helen's annual salary for the first time.

(c.i) Find the value of N. [3]

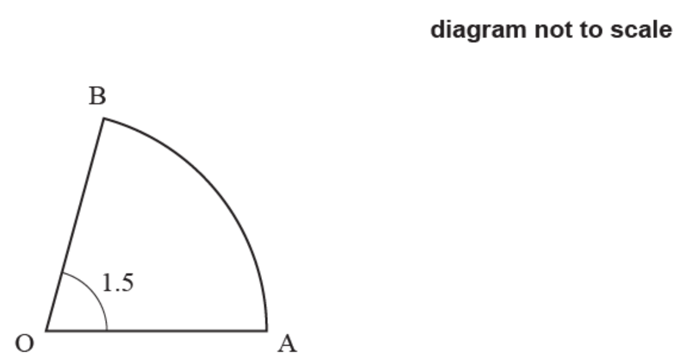
(c.ii) For the value of N found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar. [2]

(d) Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar. [4]

9. [Maximum mark: 5] 24N.1.SL.TZ2.2

Points A and B lie on a circle with centre O and radius $r\ \mathrm{cm}$, where $\mathrm{A\hat{O}B} = 1.5$ radians.

This is shown on the following diagram.



The area of sector OAB is $48\ \mathrm{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

10. [Maximum mark: 5] 24N.1.AHL.TZ0.6

(a) Solve $2x^2 - 15x + 18 < 0$. [3]

(b) The function f is defined by $f(x) = \sqrt{2x^2 - 15x + 18}$, where $x \in \mathbb{R}, x \leq k$.

Find the greatest value of k for which f^{-1} exists, justifying your answer. [2]