Kinematics P2 version 2 [23 marks]

1. [Maximum mark: 6] 19M.2.AHL.TZ2.H_6

A particle moves along a horizontal line such that at time t seconds, $t \geq 0$, its acceleration a is given by $a = 2t - 1$. When $t = 6$, its displacement s from a fixed origin O is 18.25 m. When $t = 15$, its displacement from O is 922.75 m. Find an expression for s in terms of t. [6]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to integrate a to find v ***M1***

$$v = \int a \, \mathrm{d}t = \int (2t - 1) \, \mathrm{d}t$$

$$= t^2 - t + c \quad \textbf{A1}$$

$$s = \int v \, \mathrm{d}t = \int \left(t^2 - t + c\right) \mathrm{d}t$$

$$= \frac{t^3}{3} - \frac{t^2}{2} + ct + d \quad \textbf{A1}$$

attempt at substitution of given values ***(M1)***

at $t = 6$, $18.25 = 72 - 18 + 6c + d$

at $t = 15$, $922.75 = 1125 - 112.5 + 15c + d$

solve simultaneously: ***(M1)***

$$c = -6, \; d = 0.25 \quad \textbf{A1}$$

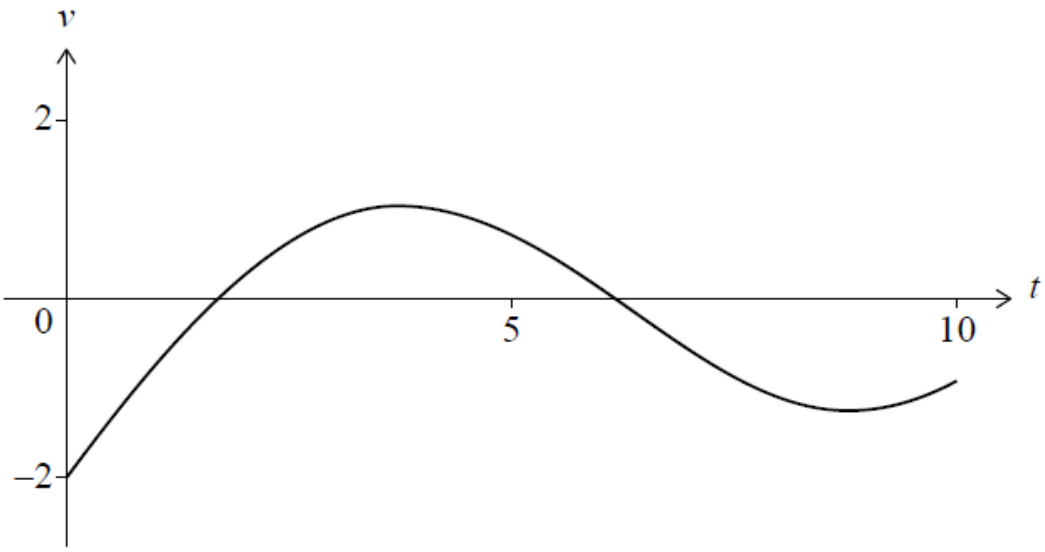
$$\Rightarrow s = \frac{t^3}{3} - \frac{t^2}{2} + -6t + \frac{1}{4}$$

[6 marks]

2. [Maximum mark: 6] 24M.2.AHL.TZ1.4

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O. For $0 \leq t \leq 10$ its velocity, v metres per second, is given by $v = 2\sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.



(a) Find the smallest value of t when the particle changes direction. [2]

Markscheme

recognition that velocity is zero ***(M1)***

$v = 2\sin(0.5t) + 0.3t - 2 = 0$

$t = 1.68694\ldots$

$t = 1.69$ ***A1***

[2 marks]

The displacement of the particle is measured in metres from O.

(b) Find the range of values of t for which the velocity is positive. [2]

Markscheme

$1.68694\ldots < t < 6.11857\ldots$

$1.69 < t < 6.12$ ***A1A1***

Note: Award ***A1*** for both values, ***A1*** for correct inequalities.

[2 marks]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

Markscheme

attempt to substitute into the total displacement formula (condone missing or incorrect limits, and absence of $\mathrm{d}\ t$) ***(M1)***

$\int_0^{10}(2\sin(0.5t) + 0.3t - 2)\ \mathrm{d}\ t$ OR $\int_0^{10} v(t)\ \mathrm{d}\ t$

$= -2.13464\ldots$

$= -2.13\,(\mathrm{m})$ ***A1***

Note: Award ***(M1)A0*** if -2.13 is followed by 2.13.

[2 marks]

3. [Maximum mark: 6] 24M.2.AHL.TZ2.4

A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v(t) = 1 + \text{e}^{-t} - \text{e}^{-\sin 2t}$ for $0 \leq t \leq 2$.

(a) Find the velocity of the particle at $t = 2$. [1]

Markscheme

$v = -0.996114\ldots$

$v = -0.996$ (ms^{-1}) ***A1***

[1 mark]

(b) Find the maximum velocity of the particle. [2]

Markscheme

considers $v'(t) = 0$ ***(M1)***

$t = 0.405833\ldots$

$v_{\max} = 1.18230\ldots$

$v_{\max} = 1.18$ (ms^{-1}) ***A1***

[2 marks]

(c) Find the acceleration of the particle at the instant it changes direction. [3]

Markscheme

recognizes that the particle changes direction when $v = 0$ ***(M1)***

Note: Award ***(M1)*** for $t = 1.65840\ldots$ seen.

finds acceleration for their value of t for which $v(t) = 0$ ***(M1)***

$v\prime(1.65840\ldots)$

$a = -2.53487\ldots$

$a = -2.53$ (ms^{-2}) ***A1***

[3 marks]

4. [Maximum mark: 5] 23N.2.AHL.TZ1.4

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 4.3\sin\left(\sqrt{3t+5}\right)$, where $0 \leq t \leq 10$.

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

Markscheme

recognizing at rest when $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ OR s is a minimum ***(M1)***

$q = 5.73553\ldots$

$= 5.74$ ***A1***

Note: If no working shown, award ***(M1)A0*** for $q = 5.7$ (2sf).

[2 marks]

(b) Find the total distance that the particle travels in the first q seconds. [3]

Markscheme

METHOD 1

recognizing that integral of $v(t)$ is required ***(M1)***

$\int_0^{5.73\ldots} |v(t)|\,\mathrm{d}t$ OR $\int_0^{5.73\ldots} \left|\frac{\mathrm{d}}{\mathrm{d}t}s(t)\right|\mathrm{d}t$ OR $\left|\int_0^{5.73\ldots} v(t)\,\mathrm{d}t\right|$ OR $-\int_0^{5.73\ldots} v(t)\,\mathrm{d}t$ ***(A1)***

Note: Condone absence of $\mathrm{d}t$.

Only accept $\left|\int_0^q v(t)\,\mathrm{d}t\right|$ if their value of q does not result in the particle changing direction in the first q seconds.

$= 7.68302\ldots$

$= 7.68\ (\mathrm{m})$ ***A1***

Note: Special Cases:
Award a maximum of ***(M1)(A1FT)A0FT*** if the candidate obtains $q = 1.62320\ldots$ in part (a), and uses that value to find the total distance to be $3.38302\ldots$ ($3.37644\ldots$ from 3sf).

Award ***(M1)(A0)A1*** if the candidate writes $\int_0^{5.73\ldots} v(t)\,\mathrm{d}t$ followed by the correct answer.

METHOD 2

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum ***(M1)***

initial displacement is $3.38302\ldots$ AND at minimum is -4.3 ***(A1)***
total distance travelled $= 3.38302\ldots - (-4.3)$

$= 7.68302\ldots$

$= 7.68\ (\text{m})$ ***A1***

Note: If no working shown, award ***(M1)(A0)A0*** for 7.7 (2sf).

[3 marks]

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