

## Maclaurin [53 marks]

1. [Maximum mark: 11]

The function  $f$  is defined by  $f(x) = \arcsin(2x)$ , where  $-\frac{1}{2} \leq x \leq \frac{1}{2}$ .

(a) By finding a suitable number of derivatives of  $f$ , find the first two non-zero terms in the Maclaurin series for  $f$ . [8]

(b) Hence or otherwise, find  $\lim_{x \rightarrow 0} \frac{\arcsin(2x) - 2x}{(2x)^3}$ . [3]

2. [Maximum mark: 21]

The function  $f$  is defined by  $f(x) = e^{\sin x}$ .

(a) Find the first two derivatives of  $f(x)$  and hence find the Maclaurin series for  $f(x)$  up to and including the  $x^2$  term. [8]

(b) Show that the coefficient of  $x^3$  in the Maclaurin series for  $f(x)$  is zero. [4]

(c) Using the Maclaurin series for  $\arctan x$  and  $e^{3x} - 1$ , find the Maclaurin series for  $\arctan(e^{3x} - 1)$  up to and including the  $x^3$  term. [6]

(d) Hence, or otherwise, find  $\lim_{x \rightarrow 0} \frac{f(x) - 1}{\arctan(e^{3x} - 1)}$ . [3]

3. [Maximum mark: 21]

The function  $f$  is defined by  $f(x) = e^x \sin x$ , where  $x \in \mathbb{R}$ .

The function  $g$  is defined by  $g(x) = e^x \cos x$ , where  $x \in \mathbb{R}$ .

- (a) Find the Maclaurin series for  $f(x)$  up to and including the  $x^3$  term. [4]
- (b) Hence, find an approximate value for  $\int_0^1 e^{x^2} \sin(x^2) \, dx$ . [4]
- (c) Show that  $g(x)$  satisfies the equation  $g''(x) = 2(g'(x) - g(x))$ . [4]
- (d) Hence, deduce that  $g^{(4)}(x) = 2(g'''(x) - g''(x))$ . [1]
- (e) Using the result from part (c), find the Maclaurin series for  $g(x)$  up to and including the  $x^4$  term. [5]
- (f) Hence, or otherwise, determine the value of  $\lim_{x \rightarrow 0} \frac{e^x \cos x - 1 - x}{x^3}$ . [3]