Revision (GDC) [150 marks]

1. [Maximum mark: 6] SPM.1.SL.TZ0.5

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}, t \geq 0$, where t is the number of days since the start of the migration season.

(a.i) Find the population of the Bulbul birds at the start of the migration season. [1]

(a.ii) Find the population of the Bulbul birds after 5 days. [2]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

2. [Maximum mark: 4] SPM.1.SL.TZ0.8

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre (W m^{-2}). The intensity level is given by the following formula.

$$L = 10 \log_{10} \left(S \times 10^{12}\right), S \geq 0.$$

(a) An orchestra has a sound intensity of 6.4×10^{-3} W m^{-2}. Calculate the intensity level, L of the orchestra. [2]

(b) A rock concert has an intensity level of 112 dB. Find the sound intensity, S. [2]

3. [Maximum mark: 6] SPM.1.SL.TZ0.2

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

4. [Maximum mark: 17] SPM.2.SL.TZ0.1

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

(a.i) Find the repayment made each quarter. [3]

(a.ii) Find the total amount paid for the car. [2]

(a.iii) Find the interest paid on the loan. [2]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

(b.i) Find the amount to be borrowed for this option. [2]

(b.ii) Find the annual interest rate, r. [3]

(c) State which option Bryan should choose. Justify your answer. [2]

(d) Bryan's car depreciates at an annual rate of 25 % per year.

Find the value of Bryan's car six years after it is purchased. [3]

5. [Maximum mark: 6] EXN.1.SL.TZ0.5

The pH of a solution is given by the formula $pH = -\log_{10} C$ where C is the hydrogen ion concentration in a solution, measured in moles per litre (Ml^{-1}).

(a) Find the pH value for a solution in which the hydrogen ion concentration is 5.2×10^{-8}. [2]

(b.i) Write an expression for C in terms of pH. [2]

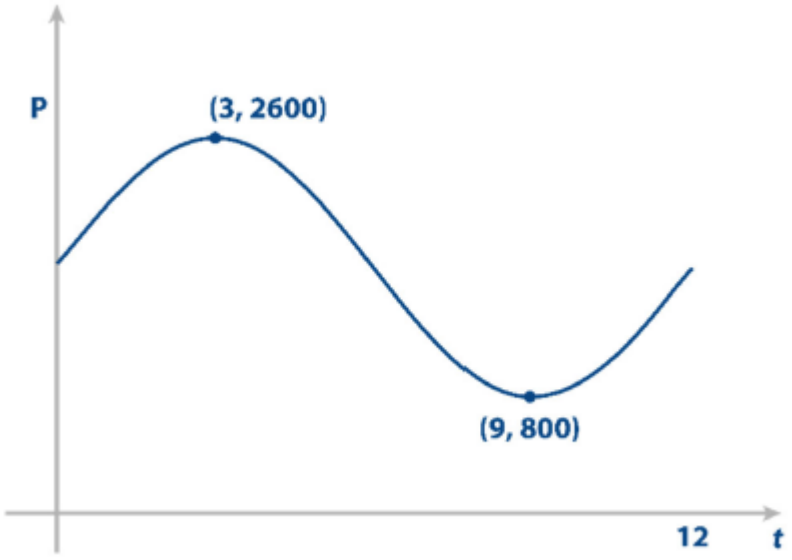
(b.ii) Find the hydrogen ion concentration in a solution with $pH\ 4.2$. Give your answer in the form $a \times 10k$ where $1 \leq a < 10$ and k is an integer. [2]

6. [Maximum mark: 7] EXN.1.SL.TZ0.6

The size of the population (P) of migrating birds in a particular town can be approximately modelled by the equation $P = a\sin(bt) + c, \quad a, b, c \in \mathbb{R}^+$, where t is measured in months from the time of the initial measurements.

In a 12 month period the maximum population is 2600 and occurs when $t = 3$ and the minimum population is 800 and occurs when $t = 9$.

This information is shown on the graph below.



(a.i) Find the value of a. [2]

(a.ii) Find the value of b. [2]

(a.iii) Find the value of c. [1]

(b) Find the value of t at which the population first reaches 2200. [2]

7. [Maximum mark: 7] EXN.1.SL.TZ0.9

Sophia pays $\$200$ into a bank account at the end of each month. The annual interest paid on money in the account is 3.1% which is compounded monthly.

(a) Find the value of her investment after a period of 5 years. [3]

The average rate of inflation per year over the 5 years was 2%.

(b) Find an approximation for the real interest rate for the money invested in the account. [2]

(c) Hence find the real value of Sophia's investment at the end of 5 years. [2]

8. [Maximum mark: 6] EXM.1.SL.TZ0.6

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of $4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

(a) Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal. [3]

(b) Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund.

Calculate the amount Yejin needs to save each month, to meet her retirement goal. [3]

9. [Maximum mark: 15] EXM.2.SL.TZ0.2

Sophie is planning to buy a house. She needs to take out a mortgage for $120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay $1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

(a.i) Calculate the monthly repayment using option 1. [2]

(a.ii) Calculate the total amount Sophie would pay, using option 1. [2]

(b.i) Calculate the number of months it will take to repay the mortgage using option 2. [3]

(b.ii) Calculate the total amount Sophie would pay, using option 2. [2]

Give a reason why Sophie might choose

(c.i) option 1. [1]

(c.ii) option 2. [1]

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

(d.i) Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years. [2]

(d.ii) Hence calculate her monthly repayment for the final 10 years. [2]

10. [Maximum mark: 13] EXM.2.SL.TZ0.3

Urvashi wants to model the height of a moving object. She collects the following data showing the height, h metres, of the object at time t seconds.

t (seconds)	2	5	7
h (metres)	34	38	24

She believes the height can be modeled by a quadratic function, $h(t) = at^2 + bt + c$, where $a,\ b,\ c \in \mathbb{R}$.

(a) Show that $4a + 2b + c = 34$. [1]

(b) Write down two more equations for a, b and c. [3]

(c) Solve this system of three equations to find the value of a, b and c. [4]

Hence find

(d.i) when the height of the object is zero. [3]

(d.ii) the maximum height of the object. [2]

11. [Maximum mark: 11] EXM.2.SL.TZ0.1

Paul wants to buy a car. He needs to take out a loan for $7000. The car salesman offers him a loan with an interest rate of 8%, compounded annually. Paul considers two options to repay the loan.

Option 1: Pay $200 each month, until the loan is fully repaid

Option 2: Make 24 equal monthly payments.

Use option 1 to calculate

(a.i) the number of months it will take for Paul to repay the loan. [3]

(a.ii) the total amount that Paul has to pay. [2]

Use option 2 to calculate

(b.i) the amount Paul pays each month. [2]

(b.ii) the total amount that Paul has to pay. [2]

Give a reason why Paul might choose

(c.i) option 1. [1]

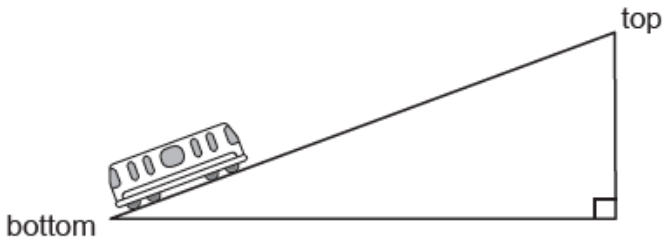
(c.ii) option 2. [1]

12. [Maximum mark: 4] 24N.1.SL.TZ1.1

One of the steepest train tracks in the world is in Tennessee, USA.

This track is $1.52\ \text{km}$ long, and the angle of elevation from the bottom of the track to the top is 36.1°.

diagram not to scale



(a) Label the diagram with the given values for the track length and the angle of elevation. [2]

(b) Find the vertical change in height from the bottom of the track to the top. [2]

13. [Maximum mark: 6] 24N.1.SL.TZ1.6

Radioactive carbon is a material that decays over time.

The mass, $m(t)$ (in nanograms), of radioactive carbon in a fossil of a plant, after t years, can be modelled by the function

$$m(t) = 120\,\mathrm{e}^{-0.000121t}$$

where t is the time since the plant died.

(a) Write down the initial mass of the radioactive carbon. [1]

(b) Find the mass of the radioactive carbon after $20\,000$ years. [2]

(c) Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]

14. [Maximum mark: 6] 24N.1.SL.TZ1.10

When Humberto retires, he invests $\$300\,000$ in an annuity fund that earns interest at a nominal rate of $3.8\,\%$ per year, compounded monthly.

Humberto then withdraws $\$2800$ at the end of every month to pay for his living expenses.

(a) Find how much is in the annuity fund after 8 years. [3]

(b) Calculate how many times Humberto is able to make these withdrawals. [3]

15. [Maximum mark: 7] 24N.1.AHL.TZ0.8

The amount of daylight, L (in hours), in London in 2024 can be modelled by

$$L = a\sin\left(b(t-c)\right) + d,$$

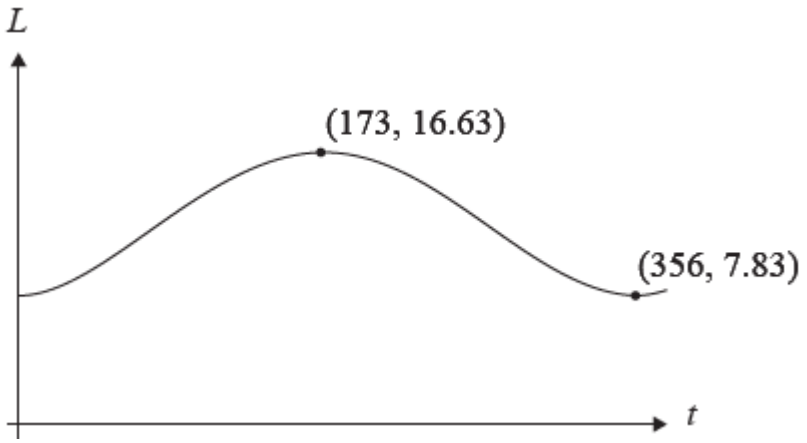
where $a, b, c, d > 0$ and t is the day of the year.

For example, day 1 = 1 January , day 2 = 2 January, and so on.

The maximum value of L is 16.63 hours on day 173 (21 June 2024).

The minimum value of L is 7.83 hours on day 356 (21 December 2024).

This information is shown in the following diagram.



Find the value of

(a) d [2]

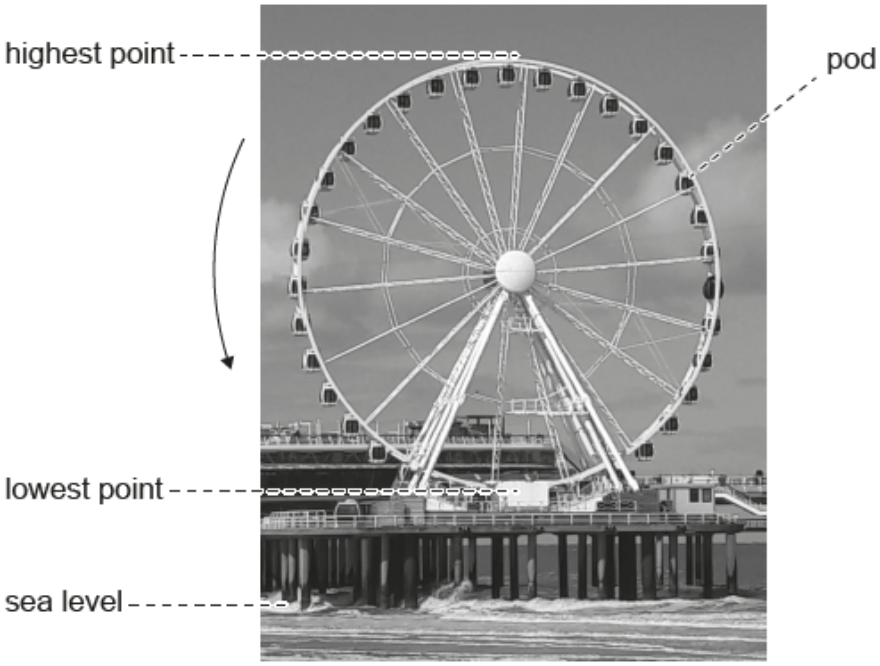
(b) a [1]

(c) b [2]

(d) c. [2]

16. [Maximum mark: 16] 24N.2.SL.TZ1.2

The Scheveningen Ferris wheel's lowest point is $8\,\text{m}$ above sea level, and its highest point is $45\,\text{m}$ above sea level.



(a.i) Show that the radius of the Ferris wheel is $18.5\,\text{m}$. [1]

(a.ii) Calculate the circumference of the Ferris wheel. [1]

There are pods, equally spaced around the wheel, that carry passengers.

(b) When the wheel rotates 10°, find the distance that a pod travels along the circumference. [3]

The height in metres, above sea level, of a particular pod is modelled by the function:

$$h(t) = a\sin(bt) + d, \text{for } a,\ b > 0,$$

where t is the time, measured in minutes.

The wheel takes 15 minutes to complete 1 revolution.

(c.i) Find the value of b. [1]

(c.ii) Find the value of d. [2]

(c.iii) Hence, write down the equation of the sinusoidal model. [2]

(d) Use the model to find the values of t when the height of this pod is $33\,\text{m}$ above sea level for $0 \leq t \leq 15$. [3]

Since the Ferris wheel opened, it has been operating for 3000 days, and each day it rotates nonstop for 8 hours.

(e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

17. [Maximum mark: 6] 24M.1.SL.TZ1.5

Maan deposited $\$100\,000$ into a savings account with a nominal annual interest rate of $I\,\%$ **compounded monthly**. At the end of the eighth year, the amount in the account had increased to $\$150\,000$.

(a) Find the value of I. [3]

Maan withdraws the $\$150\,000$ and places it in an annuity, earning a nominal annual interest rate of $6.1\,\%$ **compounded monthly**. At the end of each month, Maan will receive a payment of $\$1000$.

(b) Find the amount of money remaining in the annuity at the end of 10 years. Express your answer to the nearest dollar. [3]

18. [Maximum mark: 7] 24M.1.SL.TZ1.3

On 1 January 2025, the Faber Car Company will release a new car to global markets. The company expects to sell 40 cars in January 2025. The number of cars sold each month can be modelled by a geometric sequence where $r = 1.1$.

(a) Use this model to find the number of cars that will be sold in December 2025. [2]

(b) Use this model to find the total number of cars that will be sold in the year

(b.i) 2025. [2]

(b.ii) 2026. [3]