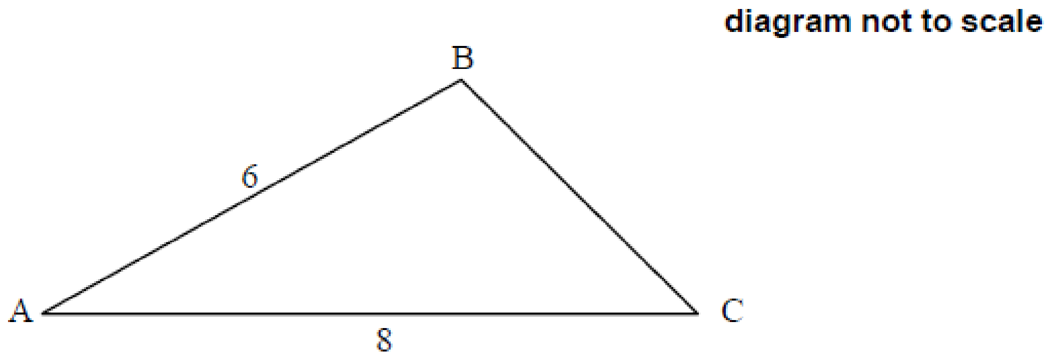
Revision (no GDC) [190 marks]

1. [Maximum mark: 5] SPM.1.SL.TZ0.1

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



(a) Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. [3]

Markscheme

valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

(b) Find the area of triangle ABC. [2]

Markscheme

$\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

area $= 4\sqrt{11}$ ***A1***

[2 marks]

2. [Maximum mark: 5] SPM.1.SL.TZ0.5

The functions f and g are defined such that $f\left(x\right)=\frac{x+3}{4}$ and $g\left(x\right)=8x+5$.

(a) Show that $\left(g\circ f\right)\left(x\right)=2x+11$. [2]

Markscheme

attempt to form composition ***M1***

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ ***A1***

$\left(g\circ f\right)\left(x\right)=2x+11$ ***AG***

[2 marks]

(b) Given that $\left(g\circ f\right)^{-1}\left(a\right)=4$, find the value of a. [3]

Markscheme

attempt to substitute 4 (seen anywhere) ***(M1)***

correct equation $a=2\times 4+11$ ***(A1)***

a = 19 ***A1***

[3 marks]

3. [Maximum mark: 5] EXN.1.SL.TZ0.2

Solve the equation $2 \ln x = \ln 9 + 4$. Give your answer in the form $x = p\mathrm{e}^q$ where $p,\ q \in \mathbb{Z}^+$. [5]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

METHOD 1

$2 \ln x - \ln 9 = 4$

uses $m \ln x = \ln x^m$ **(M1)**

$\ln x^2 - \ln 9 = 4$

uses $\ln a - \ln b = \ln \frac{a}{b}$ **(M1)**

$\ln \frac{x^2}{9} = 4$

$\frac{x^2}{9} = \mathrm{e}^4$ **A1**

$x^2 = 9\mathrm{e}^4 \Rightarrow x = \sqrt{9\mathrm{e}^4}\ \ (x > 0)$ **A1**

$x = 3\mathrm{e}^2\ \ (p = 3,\ q = 2)$ **A1**

METHOD 2

expresses 4 as $4 \ln \mathrm{e}$ and uses $\ln x^m = m \ln x$ **(M1)**

$2 \ln x = 2 \ln 3 + 4 \ln \mathrm{e}\ \ (\ln x = \ln 3 + 2 \ln \mathrm{e})$ **A1**

uses $2 \ln \mathrm{e} = \ln \mathrm{e}^2$ and $\ln a + \ln b = \ln ab$ **(M1)**

$\ln x = \ln\left(3e^2\right)$ **A1**

$x = 3e^2 \ \ (p = 3,\ q = 2)$ **A1**

METHOD 3

expresses 4 as $4\ \ln e$ and uses $m\ \ln x = \ln x^m$ **(M1)**

$\ln x^2 = \ln 3^2 + \ln e^4$ **A1**

uses $\ln a + \ln b = \ln ab$ **(M1)**

$\ln x^2 = \ln\left(3^2 e^4\right)$

$x^2 = 3^2 e^4 \Rightarrow x = \sqrt{3^2 e^4} \ \ (x > 0)$ **A1**

so $x = 3e^2 \ \ (x > 0) \ \ (p = 3,\ q = 2)$ **A1**

[5 marks]

4. [Maximum mark: 6] EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

Markscheme

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EITHER

uses $u_2 - u_1 = u_3 - u_2$ **(M1)**

$(5u_1 - 8) - u_1 = (3u_1 + 8) - (5u_1 - 8)$

$6u_1 = 24$ **A1**

OR

uses $u_2 = \frac{u_1+u_3}{2}$ **(M1)**

$5u_1 - 8 = \frac{u_1+(3u_1+8)}{2}$

$3u_1 = 12$ **A1**

THEN

so $u_1 = 4$ **AG**

[2 marks]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

Markscheme

$d = 8$ **(A1)**

uses $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ **M1**

$S_n = \frac{n}{2}(8 + 8(n-1))$ **A1**

$= 4n^2$

$= (2n)^2$ **A1**

Note: The final **A1** can be awarded for clearly explaining that $4n^2$ is a square number.

so sum of the first n terms is a square number **AG**

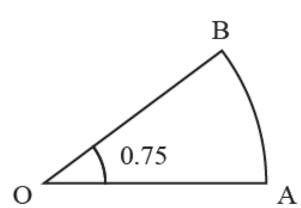
[4 marks]

5. [Maximum mark: 5] 24N.1.SL.TZ1.2

Points A and B lie on a circle with centre O and radius r cm, where $\mathrm{A\hat{O}B} = 0.75$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $6\ \mathrm{cm}^2$.

(a) Find the value of r. [3]

Markscheme

$\frac{1}{2}r^2(\theta) = 6$ OR $\frac{1}{2}r^2(0.75) = 6$ ***(A1)*** attempt to solve their equation to find r or r^2 ***(M1)***

Note: To award the ***M1***, candidate's equation must include r^2 and $\theta = 1.5$, and they must isolate r^2 or r.

$r^2 = 16$

$r = 4$ (cm) ***A1***

[3 marks]

(b) Hence, find the perimeter of sector OAB. [2]

Markscheme

evidence of summing the two radii and the arc length ***(M1)***

perimeter $= 2r + r\theta$

$= 8 + 4(0.75)$

$= 11$ (cm) ***A1***

[2 marks]

6. [Maximum mark: 6] 24N.1.SL.TZ1.6

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$. [6]

Markscheme

attempt to use $u_n = u_1 + (n-1)d$ or $S_n = \frac{n}{2}[2u_1 + (n-1)d]$ or $S_n = \frac{n}{2}[u_1 + u_n]$ to set up at least one equation in u_1 and d ***(M1)***

$14 = u_1 + 9d$ and $200 = \frac{25}{2}[2u_1 + 24d]$ ***A1***

attempt to solve their two linear equations in u_1 and d simultaneously (must eliminate one variable) ***(M1)***

$d = -2 \ (\Rightarrow u_1 = 32)$ ***(A1)***

attempt to solve $u_k = 0$ with their d (or with their d and u_1) ***(M1)***

$\Rightarrow k = 17$ ***A1***

[6 marks]

7. [Maximum mark: 14] 24N.1.SL.TZ1.8

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(a.i) $f(2)$; [2]

Markscheme

substitution of $x = 2$ in part (i) or $x = \frac{1}{8}$ in part (ii) ***(M1)***

$\log_2(8 \times 2)$ OR $\log_2(16)$

$\log_2(16) = 4$ ***A1***

[2 marks]

(a.ii) $f\left(\frac{1}{8}\right)$.

[1]

Markscheme

$f\left(\frac{1}{8}\right) = 0$ ***A1***

[1 mark]

(b) Find an expression for $f^{-1}(x)$. [4]

Markscheme

swap x and y ***(M1)***

$x = \log_2(8y)$ OR $x = 3 + \log_2 y$

attempt to write as exponential ***(M1)***

$2^x = 8y$ ***(A1)***

$\left(f^{-1}(x) =\right) \frac{2^x}{8} \quad \left(= 2^{x-3}\right)$ (accept $y = \frac{2^x}{8}$ or $y = 2^{x-3}$)
A1

[4 marks]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

Markscheme

$\frac{1}{8}$ ***A1***

[1 mark]

The graph of $y = f\left(4x^2\right)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

Markscheme

METHOD 1

$$f\left(4x^2\right) = \log_2\left(8 \times 4x^2\right)$$

attempt to use addition rule for logs ***(M1)***

$\log_2 8 + \log_2 4 + \log_2 x^2$ OR $\log_2 32 + \log_2 x^2$ (or equivalent) ***(A1)***

attempt to use exponent property for logarithms ***(M1)***

$f\left(4x^2\right) = 5 + 2\log_2 x$ (or equivalent) ***A1***

the graph of g must be vertically stretched (dilated) by a scale factor of 2 and then vertically translated (shifted) 5 units upwards. ***A2***

METHOD 2

$$f\left(4x^2\right) = \log_2\left(8 \times 4x^2\right)$$

attempt to write argument as a power ***(M1)***

$\log_2\left(32x^2\right) = \log_2\left(\left(\sqrt{32}x\right)^2\right)$ (or equivalent) ***(A1)***

attempt to use exponent property for logarithms ***(M1)***

$f\left(4x^2\right) = 2\log_2\left(\sqrt{32}x\right)$ (or equivalent) ***A1***

EITHER

the graph of g must be vertically stretched (dilated) by a scale factor of 2 and stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$. ***A1***

OR

the graph of g must be stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$ and vertically stretched (dilated) by a scale factor of 2. ***A1***

Note: In this method, the final mark is ***A1***, as the question specifically asks for a translation and a stretch.

[6 marks]

8. [Maximum mark: 16] 24M.1.AHL.TZ2.10

Consider the arithmetic sequence $a,\ p,\ q\ldots,$ where $a,\ p,\ q \neq 0$.

(a) Show that $2p - q = a$. [2]

Markscheme

attempt to find a difference ***(M1)***

$d = p - a, 2d = q - a, d = q - p$ OR
$p = a + d, q = a + 2d, q = p + d$

correct equation ***A1***

$p - a = q - p$ OR $q - a = 2(p - a)$ OR $p = \frac{a+q}{2}$ (or equivalent)

$2p - q = a$ ***AG***

[2 marks]

Consider the geometric sequence $a,\ s,\ t\ldots,$ where $a,\ s,\ t \neq 0$.

(b) Show that $s^2 = at$. [2]

Markscheme

attempt to find a ratio *(M1)*

$r = \frac{s}{a}$, $r^2 = \frac{t}{a}$, $r = \frac{t}{s}$ OR $s = ar, t = ar^2, t = sr$

correct equation **A1**

$\left(\frac{s}{a}\right)^2 = \frac{t}{a}$ OR $\frac{s}{a} = \frac{t}{s}$ (or equivalent)

$s^2 = at$ **AG**

[2 marks]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Markscheme

EITHER

$2p - 1 = s^2$ (or equivalent) **A1**

$(s^2 > 0) \Rightarrow 2p - 1 > 0$ OR $s = \sqrt{2p - 1} \Rightarrow 2p - 1 > 0$ OR $p = \frac{s^2+1}{2}$ (and $s^2 > 0$) **R1**

OR

$2p - 1 = a$ and $s^2 = a$ **A1**

($s^2 > 0,$ so) $a > 0 \Rightarrow 2p - 1 > 0$ OR $p\frac{a+1}{2}$ and $a > 0$ ***R1***

$\Rightarrow p > \frac{1}{2}$ ***AG***

Note: Do not award ***A0R1***.

[2 marks]

Consider the case where $a = 9,\ s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(d.i) arithmetic sequence; [2]

Markscheme

$9, 5, 1, -3$ ***A1A1***

Note: Award ***A1*** for each of 2nd term and 4th term

[2 marks]

(d.ii) geometric sequence. [2]

Markscheme

$9, 3, 1, \frac{1}{3}$ ***A1A1***

Note: Award ***A1*** for each of 2nd term and 4th term

[2 marks]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9,\ u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e.i) Find the common difference of the new sequence in terms of $\ln 3$. [3]

Markscheme

attempt to find the difference between two consecutive terms ***(M1)***

$d = u_2 - u_1 = 5 + \ln 3 - 9 - \ln 9$ OR
$d = u_3 - u_2 = 1 + \ln 1 - 5 - \ln 3$

$\ln 9 = 2\ln 3$ OR $\ln 1 = 0$ OR
$\ln 3 - \ln 9 = \ln \frac{1}{3} \left(= \ln 3^{-1} = -\ln 3\right)$ (seen anywhere) ***(A1)***

$d = -4 - \ln 3$ ***A1***

[3 marks]

(e.ii) Show that $\sum_{i=1}^{10} = -90 - 25\ln 3$. [3]

Markscheme

METHOD 1

attempt to substitute first term and their common difference into S_{10} ***(M1)***

$\frac{10}{2}(2(9 + \ln 9) + 9(-4 - \ln 3))$ OR
$\frac{10}{2}(2(9 + 2\ln 3) + 9(-4 - \ln 3))$ (or equivalent) ***A1***

$= 5(-18 - 5 \ln 3)$ (or equivalent in terms of $\ln 3$) ***A1***

$\sum_{i=1}^{10} u_i = -90 - 25 \ln 3$ ***AG***

METHOD 2

$u_{10} = 9 + \ln 9 + 9(-4 - \ln 3)(= -27 + \ln 9 - 9 \ln 3)$

attempt to substitute first term and their u_{10} into S_{10} ***(M1)***

$\frac{10}{2}(2(9 + \ln 9) + 9(-4 - \ln 3))$ OR
$\frac{10}{2}(9 + \ln 9 - 27 + \ln 9 - 9 \ln 3)$ OR

$\frac{10}{2}(2(9 + 2 \ln 3) + 9(-4 - \ln 3))$ OR
$\frac{10}{2}(9 + \ln 9 - 27 - 7 \ln 3)$ (or equivalent) ***A1***

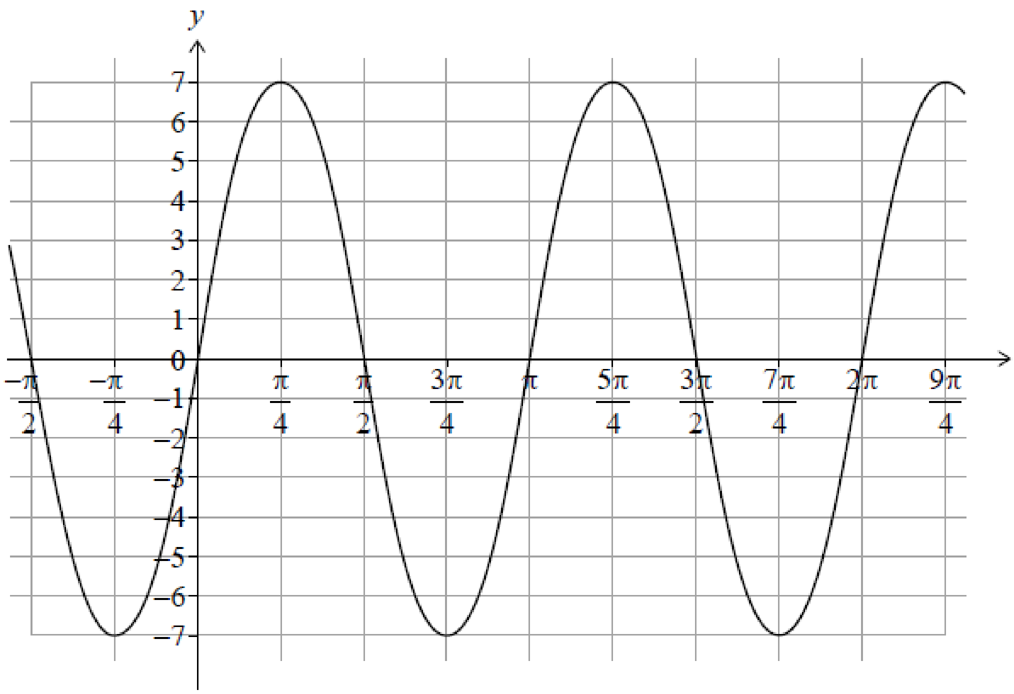
$= 5(-18 - 5 \ln 3)$ (or equivalent in terms of $\ln 3$) ***A1***

$\sum_{i=1}^{10} u_i = -90 - 25 \ln 3$ ***AG***

[3 marks]

9. [Maximum mark: 7] 23N.1.SL.TZ1.1

Consider the function $f(x) = a \sin (bx)$ with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.



(a) Write down the value of a. [1]

Markscheme

$a = 7$ ***A1***

[1 mark]

(b.i) Write down the period of f. [1]

Markscheme

period = π ***A1***

[1 marks]

(b.ii) Hence, find the value of b. [2]

Markscheme

$b = \frac{2\pi}{\pi}$ OR $\pi = \frac{2\pi}{\mathrm{b}}$ ***(A1)***

$= 2$ ***A1***

[2 marks]

(c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

Markscheme

substituting $\frac{\pi}{12}$ into their $f(x)$ ***(M1)***

$f\left(\frac{\pi}{12}\right) = 7\sin\left(\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ ***(A1)***

$= \frac{7}{2}$ **A1**

[3 marks]

10. [Maximum mark: 5] 23N.1.SL.TZ1.2

Consider the functions $f(x) = x + 2$ and $g(x) = x^2 - k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

Markscheme

attempt to form $(g \circ f)(x)$ ***(M1)***

$((g \circ f)(x)) = (x+2)^2 - k^2 \ \left(= x^2 + 4x + 4 - k^2\right)$
A1

[2 marks]

(b) Given that $(g \circ f)(4) = 11$, find the possible values of k. [3]

Markscheme

substituting $x = 4$ into their $(g \circ f)(x)$ and setting their expression $= 11$ ***(M1)***

$(4+2)^2 - k^2 = 11$ OR $4^2 + 4(4) + 4 - k^2 = 11$

$k^2 = 25$ OR $-k^2 = -25$ ***(A1)***

$k = \pm 5$ ***A1***

[3 marks]

11. [Maximum mark: 7] 23N.1.SL.TZ1.4

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_5 = 65$ and $S_6 = 96$.

(a) Find the value of p and the value of q. [5]

Markscheme

METHOD 1

attempt to form at least one equation, using either S_5 or S_6 ***(M1)***

$65 = 25p - 5q$ $(13 = 5p - q)$ **and** $96 = 36p - 6q$ $(16 = 6p - q)$ ***(A1)*** valid attempt to solve simultaneous linear equations in *p* and *q* and by substituting or eliminating one of the variables. ***(M1)***

$p = 3\ ,\ q = 2$ ***A1A1***

Note: If candidate does not explicitly state their values of *p* and *q*, but gives $S_n = 3n^2\ - 2n$, award final two marks as ***A1A0***.

METHOD 2

attempt to form at least one equation, using either S_5 or S_6 ***(M1)***

$65 = \frac{5}{2}(2u_1 + 4d)\ (26 = 2u_1 + 4d)$ **and**
$96 = 3(2u_1 + 5d)\ (32 = 2u_1 + 5d)$ ***(A1)***

valid attempt to solve simultaneous linear equations in u_1 and d by substituting or eliminating one of the variables. ***(M1)***

$u_1 = 1,\ d = 6$ ***A1***

$S_n = \frac{n}{2}(2 + 6(n-1)) = 3n^2 - 2n$

$p = 3$ and $q = 2$ ***A1***

Note: If candidate does not explicitly state their values of *p* and *q*, do not award the final mark.

[5 marks]

(b) Find the value of u_6. [2]

Markscheme

$u_6 = S_6 - S_5$ OR substituting their values of u_1 and d into $u_6 = u_1\ +\ 5d$

OR substituting their value of u_1 into $96 = \frac{6}{2}(u_1 + u_6)$ ***(M1)***

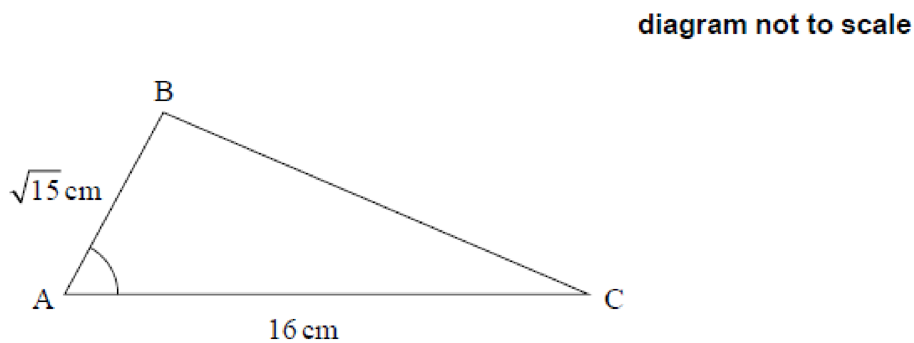
$(u_6 =)96 - 65$ OR $(u_6 =)1 + 5 \times 6$ OR $96 = 3(1 + u_6)$

$= 31$ ***A1***

[2 marks]

12. [Maximum mark: 6] 23N.1.SL.TZ1.5

In the following triangle ABC, $\mathrm{AB} = \sqrt{15\text{ cm}}$, $\mathrm{AC} = 16\text{ cm}$ and $\cos \mathrm{B\hat{A}C} = \frac{1}{4}$.



Find the area of triangle ABC. [6]

Markscheme

METHOD 1

EITHER

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\left(\sqrt{4^2 - 1^2} =\right)\sqrt{15}$ ***(A1)***

OR

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ ***(M1)***

$\sin^2 B\hat{A}C = 1 - \left(\frac{1}{4}\right)^2$ *(A1)*

THEN

$\sin B\hat{A}C = \frac{\sqrt{15}}{4}$ (may be seen in area formula) *A1*

attempt to use 'Area$=\frac{1}{2}ab\sin C$' (must include their calculated value of $\sin B\hat{A}C$) *(M1)*

$= \frac{1}{2} \times 16 \times \sqrt{15} \times \frac{\sqrt{15}}{4}$ *(A1)*

$= 30 \left(\text{cm}^2\right)$ *A1*

[6 marks]

Method 2

attempt to find the perpendicular height of triangle BAC *(M1)*

EITHER

$\text{height} = \sqrt{15} \times \sin B\hat{A}C$

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$
(M1)

$\text{height} = \sqrt{15} \times \sqrt{1 - \left(\frac{1}{4}\right)^2}$ *(A1)*

$= \sqrt{15} \times \frac{\sqrt{15}}{4} \left(= \frac{15}{4}\right)$ (may be seen in area formula) *A1*

OR

$\text{adjacent} = \frac{\sqrt{15}}{4}$ *(A1)*

attempt to use Pythagoras' theorem in a right-angled triangle. *(M1)* height $= \sqrt{15 - \frac{15}{16}} \left(= \frac{15}{4}\right)$ (may be seen in area formula)

A1

THEN

attempt to use 'Area = $\frac{1}{2}$ base $\times$ height' (must include their calculated value for height) ***(M1)***

$= \frac{1}{2} \times 16 \times \frac{15}{4}$

= $30 \left(\text{cm}^2\right)$ ***A1***

[6 marks]

13. [Maximum mark: 15] 23N.1.SL.TZ1.8

The functions f and g are defined by

$f(x) = \ln(2x - 7)$, where $x > \frac{7}{2}$

$g(x) = 2 \ln x - \ln d$, where $x > 0,\ d \in \mathbb{R}^+$.

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

Markscheme

$x = 0$ ***A1***

[1 mark]

The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b.i) Show that, at the points of intersection, $x^2 - 2dx + 7d = 0$.

[4]

Markscheme

setting $\ln\,(2x-7) = 2\ln\,x - \ln\,d$ ***M1***

attempt to use power rule ***(M1)***

$2\ln\,x = \ln\,x^2$ (seen anywhere)

attempt to use product/quotient rule for logs ***(M1)***

$\ln\,(2x-7) = \ln\,\frac{x^2}{d}$ OR $\ln\,\frac{x^2}{2x-7} = \ln\,d$ OR
$\ln\,(2x-7)d = \ln\,x^2$

$\frac{x^2}{d} = 2x-7$ OR $\frac{x^2}{2x-7} = d$ OR $(2x-7) = x^2$ ***A1***

$x^2 - 2dx + 7d = 0$ ***AG***

[4 marks]

(b.ii) Hence, show that $d^2 - 7d > 0.$ [3]

Markscheme

discriminant = $(-2d)^2 - 4 \times 7d$ ***(A1)***

recognizing discriminant > 0 ***(M1)***

$(2d)^2 - 4 \times 7d > 0$ OR $4d^2 - 28d > 0$ ***A1***

$d^2 - 7d > 0$ ***AG***

[3 marks]

(b.iii) Find the range of possible values of *d*. [2]

Markscheme

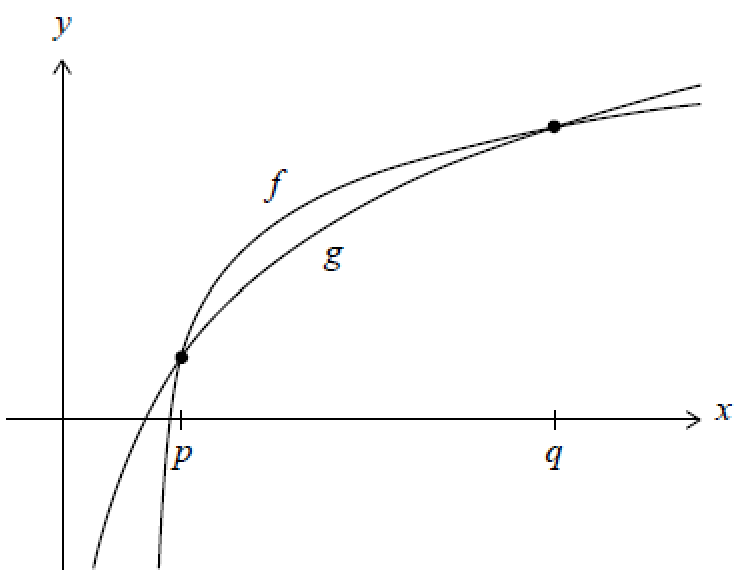
setting $d(d-7)>0$ OR $d(d-7)=0$ OR sketch graph OR sign test OR $d^2>7d$ ***(M1)***

$d<0$ or $d>7,$ but $d\in\mathbb{R}^+$

$d>7$ (or $]7,\infty[$) ***A1***

[2 marks]

The following diagram shows parts of the graph $y=f(x)$ and $y=g(x)$.



The graphs intersect at $x=p$ and $x=q$, where $p<q$.

(c) In the case where $d=10$, find the value of $q-p$. Express your answer in the form $a\sqrt{b}$, where $a,\ b\in\mathbb{Z}^+$. [5]

Markscheme

$x^2-20x+70\ (=0)$ ***A1***

attempting to solve their 3 term quadratic equation ***(M1)***

$\left((x-10)^2-30=0\right)$ or $\left((\text{x} =)\frac{20\pm\sqrt{(-20)^2-4\times1\times70}}{2}\right)$

$x=10-\sqrt{30}(=p)$ or $x=10+\sqrt{30}(=q)$ ***(A1)***

subtracting their values of x ***(M1)***

distance $=2\sqrt{30}\left(\text{or }\sqrt{120}\right)$ **A1**

$(a=2,\ b=30)\ (\text{or }\ a=1,\ b=120)$

[5 marks]

14. [Maximum mark: 7] 23M.1.SL.TZ1.2

The function f is defined by $f(x)=\frac{7x+7}{2x-4}$ for $x\in\mathbb{R}, x\neq 2$.

(a) Find the zero of $f(x)$. [2]

Markscheme

recognizing $f(x)=0$ ***(M1)***

$x=-1$ ***A1***

[2 marks]

(b) For the graph of $y=f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

Markscheme

$x = 2$ (must be an equation with x) ***A1***

[1 mark]

(b.ii) the horizontal asymptote. [1]

Markscheme

$y = \frac{7}{2}$ (must be an equation with y) ***A1***

[1 mark]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

Markscheme

EITHER

interchanging x and y ***(M1)***

$2xy - 4x = 7y + 7$

correct working with y terms on the same side: $2xy - 7y = 4x + 7$ ***(A1)***

OR

$2yx - 4y = 7x + 7$

correct working with x terms on the same side: $2yx - 7x = 4y + 7$ ***(A1)***

interchanging x and y OR making x the subject $x = \frac{4y+7}{2y-7}$ ***(M1)***

THEN

$f^{-1}(x) = \frac{4x+7}{2x-7}$ (or equivalent) $\left(x \neq \frac{7}{2}\right)$ ***A1***

[3 marks]

15. [Maximum mark: 14] 23M.1.SL.TZ1.8

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

Markscheme

recognition that $n = 5$ ***(M1)***

$S_5 = 45$ ***A1***

[2 marks]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

Markscheme

METHOD 1

recognition that $S_5 + u_6 = S_6$ ***(M1)***

$u_6 = 15$ ***A1***

METHOD 2

recognition that $60 = \frac{6}{2}\left(S_1 + u_6\right)$ ***(M1)***

$60 = 3(5 + u_6)$

$u_6 = 15$ ***A1***

METHOD 3

substituting their u_1 and d values into $u_1 + (n-1)d$ ***(M1)***

$u_6 = 15$ ***A1***

[2 marks]

(b) Find u_1. [2]

Markscheme

recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their u_6 into S_6 ***(M1)***

OR equations for S_5 and S_6 in terms of u_1 and d

$1 + 4$ OR $60 = \frac{6}{2}(U_1 + 15)$

$u_1 = 5$ ***A1***

[2 marks]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Markscheme

EITHER

valid attempt to find d (may be seen in (a) or (b)) ***(M1)***

$d = 2$ ***(A1)***

OR

valid attempt to find $S_n - S_{n-1}$ ***(M1)***

$n^2 + 4n - (n^2 - 2n + 1 + 4n - 4)$ ***(A1)***

OR

equating $n^2 + 4n = \frac{n}{2}(5 + u_n)$ ***(M1)***

$2n + 8 = 5 + u_n$ (or equivalent) ***(A1)***

THEN

$u_n = 5 + 2(n - 1)$ OR $u_n = 2n + 3$ ***A1***

[3 marks]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

Markscheme

recognition that $v_2 r^2 = v_4$ OR $(v_3)^2 = v_2 \times v_4$ ***(M1)***

$r^2 = 3$ OR $v_3 = (\pm) 5\sqrt{3}$ ***(A1)***

$r = \pm\sqrt{3}$ ***A1***

Note: If no working shown, award ***M1A1A0*** for $\sqrt{3}$.

[3 marks]

(e) Given that $v_{99} < 0$, find v_5. [2]

Markscheme

recognition that r is negative ***(M1)***

$v_5 = -15\sqrt{3} \ \left(= -\frac{45}{\sqrt{3}}\right)$ ***A1***

[2 marks]

16. [Maximum mark: 7] 23M.1.SL.TZ2.6

The functions f and g are defined for $x \in \mathbb{R}$ by

$f(x) = ax + b$, where $a, b \in \mathbb{Z}$

$g(x) = x^2 + x + 3.$

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15.$

[7]

Markscheme

attempts to form $(g \circ f)(x)$ ***(M1)***

$[f(x)]^2 + f(x) + 3$ OR $(ax + b)^2 + ax + b + 3$

$a^2x^2 + 2abx + b^2 + ax + b + 3 (= 4x^2 - 14x + 15)$ ***(A1)***

equates their corresponding terms to form at least one equation ***(M1)***

$a^2x^2 = 4x^2$ OR $a^2 = 4$ OR $2abx + ax = -14x$ OR $2ab + a = -14$ OR $b^2 + b + 3 = 15$

$a = \pm 2$ (seen anywhere) ***A1***

attempt to use $2ab + a = -14$ to pair the correct values (seen anywhere) ***(M1)***

$f(x) = 2x - 4$ (accept $a = 2$ with $b = -4$), $f(x) = -2x + 3$ (accept $a = -2$ with $b = 3$) ***A1A1***

[7 marks]

17. [Maximum mark: 5] 23M.1.SL.TZ2.3

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}, x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(a.i) the vertical asymptote; [1]

Markscheme

$x = 2$ ***A1***

[1 mark]

(a.ii) the horizontal asymptote. [1]

Markscheme

$y = 1$ ***A1***

[1 mark]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(b.i) the y-axis; [1]

Markscheme

$\left(0, \frac{3}{2}\right)$ ***A1***

[1 mark]

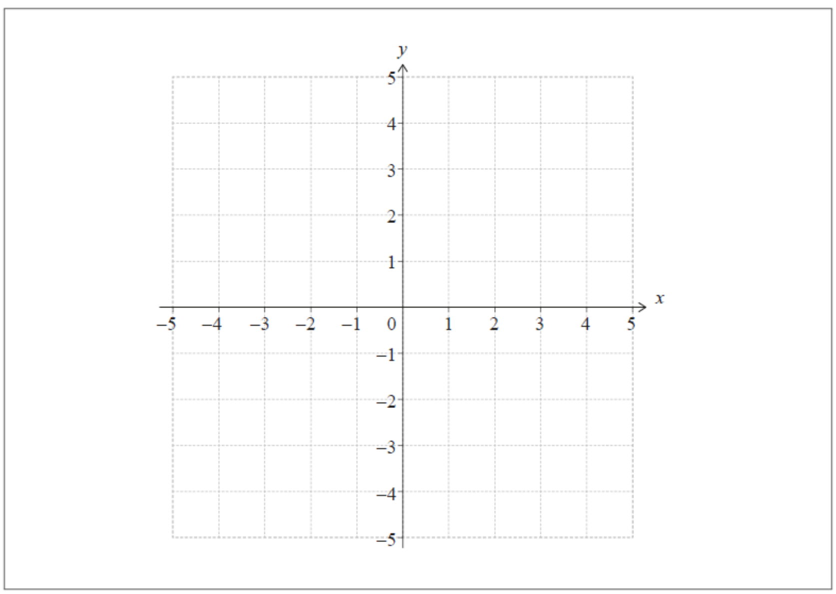
(b.ii) the x-axis. [1]

Markscheme

$(3, 0)$ ***A1***

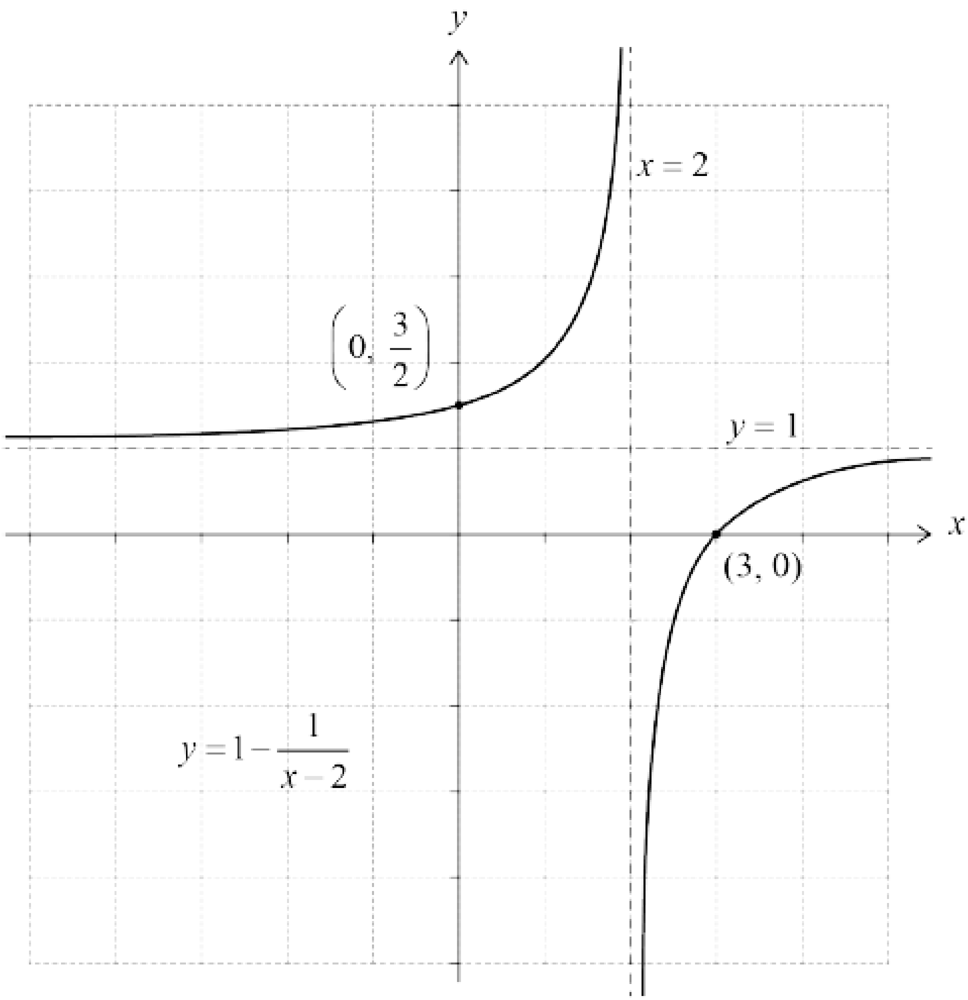
[1 mark]

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).



[1]

Markscheme



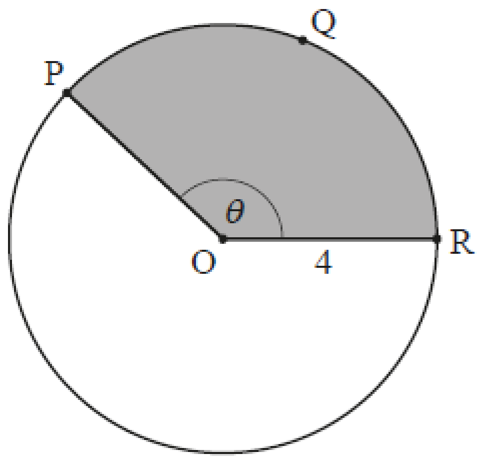
two correct branches with correct asymptotic behaviour and intercepts clearly shown ***A1***

[1 mark]

18. [Maximum mark: 6] 23M.1.SL.TZ2.1

The following diagram shows a circle with centre O and radius $4\ \mathrm{cm}$.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\mathrm{P\hat{O}R} = \theta$, where θ is measured in radians.

The length of arc PQR is $10\ \mathrm{cm}$.

(a) Find the perimeter of the shaded sector. [2]

Markscheme

attempts to find perimeter ***(M1)***

$\mathrm{arc} + 2 \times \mathrm{radius}$ OR $10 + 4 + 4$

$= 18\,(\mathrm{cm})$ ***A1***

[2 marks]

(b) Find θ. [2]

Markscheme

$10 = 4\theta$ ***(A1)***

$\theta = \frac{10}{4}\left(= \frac{5}{2}, 2.5\right)$ ***A1***

[2 marks]

(c) Find the area of the shaded sector. [2]

Markscheme

area $= \frac{1}{2}\left(\frac{10}{4}\right)\left(4^2\right)\ \left(= 1.25 \times 16\right)$ ***(A1)***

$= 20\left(\mathrm{cm}^2\right)$ ***A1***

[2 marks]

19. [Maximum mark: 15] 22N.1.SL.TZ0.8

Calculate the value of each of the following logarithms:

(a.i) $\log_2 \frac{1}{16}$. [2]

Markscheme

valid approach to find the required logarithm ***(M1)***

$2^x = \frac{1}{16}$ OR $2^x = 2^{-4}$ OR $\frac{1}{16} = 2^{-4}$ OR $\log_2 1 - \log_2 16$

$\log_2 \frac{1}{16} = -4$ ***A1***

[2 marks]

(a.ii) $\log_9 3$. [2]

Markscheme

valid approach to find the required logarithm *(M1)*

$9^x = 3$ OR $3^{2x} = 3$ OR $3 = 9^{\frac{1}{2}}$ OR $\frac{\log_3 3}{\log_3 9}$

$\log_9 3 = \frac{1}{2}$ ***A1***

[2 marks]

(a.iii) $\log_{\sqrt{3}} 81$. [3]

Markscheme

$\left(\sqrt{3}\right)^x = 81$ OR $\frac{\log_3 81}{\log_3 \sqrt{3}}$ *(A1)*

$(3)^{\frac{x}{2}} = 3^4$ OR $\frac{x}{2} = 4$ OR $\frac{4}{\frac{1}{2}}$ *(A1)*

$x = 8$ ***A1***

[3 marks]

It is given that $\log_{ab} a = 3$, where $a,\ b \in \mathbb{R}^+,\ ab \neq 1$.

(b.i) Show that $\log_{ab} b = -2$. [4]

Markscheme

Note: There are many valid approaches to the question, and the steps may be seen in different ways. Some possible methods are given here, but candidates may use a combination of one or more of these methods.

In all methods, the final ***A*** mark is awarded for working which leads directly to the ***AG***.

METHOD 1

$(ab)^3 = a$ ***(A1)***

attempt to isolate b or a power of b ***(M1)***

correct working ***(A1)***

$b = \frac{a}{a^3b^2}$ OR $b^3 = a^{-2}$ OR $b^{-1} = (ab)^2$ OR $b^3 = \frac{1}{a^2}$

$b = \frac{1}{a^2b^2}$ OR $b = (ab)^{-2}$ OR $3 \log_{ab} b = -2 \log_{ab} a$ OR $-\log_{ab} b = 2 \log_{ab} ab$ ***A1***

$\log_{ab} b = -2$ ***AG***

METHOD 2

$(ab)^3 = a$ ***(A1)***

taking logarithm to base ab on both sides ***(M1)***

$\log_{ab} (ab)^3 = \log_{ab} a$ OR $\log_{ab} a^3b^3 = \log_{ab} a$

correct application of log rules leading to equation in terms of $\log_{ab}$ ***(A1)***

$3 \log_{ab} a + 3 \log_{ab} b = \log_{ab} a$ OR $3 \log_{ab} b = -2 \log_{ab} a$ OR $\log_{ab} b^3 = \log_{ab} a^{-2}$

$\log_{ab} b = \log_{ab} a^{-\frac{2}{3}}$ OR $\log_{ab} b = -\frac{2}{3}\log_{ab} a$ OR $\log_{ab} b = -\frac{2}{3}(3)$ ***A1***

$\log_{ab} b = -2$ ***AG***

Note: Candidates may substitute $\log_{ab} a = 3$ at any point in their working.

METHOD 3

$\log_{ab} a = 3$

writing in terms of base a ***(M1)***

$\frac{\log_a a}{\log_a ab}(= 3)$

correct application of log rules ***(A1)***

$\frac{\log_a a}{\log_a a + \log_a b}(= 3)$ OR $\frac{1}{1+\log_a b}(= 3)$ OR $3\log_a b = -2$ OR $\log_a b = -\frac{2}{3}$

writing $\log_{ab} b$ in terms of base a ***(A1)***

$\log_{ab} b = \frac{\log_a b}{\log_a a + \log_a b}$

correct working ***A1***

$\log_{ab} b = \frac{-\frac{2}{3}}{1-\frac{2}{3}}$ OR $\frac{\left(-\frac{2}{3}\right)}{\left(\frac{1}{3}\right)}$

$\log_{ab} b = -2$ ***AG***

METHOD 4

$\log_{ab} ab = 1$ ***A2***

$\log_{ab} a + \log_{ab} b = 1$ ***(A1)***

$3 + \log_{ab} b = 1$ ***A1***

$\log_{ab} b = -2$ ***AG***

[4 marks]

(b.ii) Hence find the value of $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}$. [4]

Markscheme

applying the quotient rule or product rule for logs

$\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} - \log_{ab} \sqrt{b}$ OR
$\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} + \log_{ab} \frac{1}{\sqrt{b}}$ ***(A1)***

correct working ***(A1)***

$= \frac{1}{3}\log_{ab} a - \frac{1}{2}\log_{ab} b$ OR $\log_{ab} ab - \log_{ab} \sqrt{b}$

$= \frac{1}{3} \cdot 3 - \frac{1}{2}(-2)$ ***(A1)***

$= 2$ ***A1***

Note: Award ***A1A0A0A1*** for a correct answer with no working.

[4 marks]

20. [Maximum mark: 16] 22N.1.SL.TZ0.7

(a) The graph of a quadratic function f has its vertex at the point $(3,\ 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f\left(x\right) = a(x - h)^2 + k$. [3]

Markscheme

correct substitution of $h = 3$ and $k = 2$ into $f(x)$ ***(A1)***

$f(x) = a(x - 3)^2 + 2$

correct substitution of $(5,\ 0)$ ***(A1)***

$0 = a(5 - 3)^2 + 2\ \left(a = -\frac{1}{2}\right)$

Note: The first two A marks are independent.

$f(x) = -\frac{1}{2}(x - 3)^2 + 2$ ***A1***

[3 marks]

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p,\ t \in \mathbb{R},\ p \neq 0$.

In the case where $g(-3) = g(1) = 4$,

(b.i) find the value of p and the value of t. [4]

Markscheme

METHOD 1

correct substitution of $(1,\ 4)$ ***(A1)***

$p+(t-1)-p=4$

$t=5$ ***A1***

substituting their value of t into $9p-3(t-1)-p=4$ ***(M1)***

$8p-12=4$

$p=2$ ***A1***

METHOD 2

correct substitution of ONE of the coordinates $(-3,\ 4)$ or $(1,\ 4)$ ***(A1)***

$9p-3(t-1)-p=4$ OR $p+(t-1)-p=4$

valid attempt to solve their two equations ***(M1)***

$p=2,\ t=5$ ***A1A1***

$\left(g(x)=2x^2+4x-2\right)$

[4 marks]

(b.ii) find the range of g. [3]

Markscheme

attempt to find the x-coordinate of the vertex ***(M1)***

$x=\frac{-3+1}{2}(=-1)$ OR $\frac{-4}{2\times2}$ OR $4x+4=0$ OR $2(x+1)^2-4$

y-coordinate of the vertex $=-4$ ***(A1)***

correct range ***A1***

$[-4, +\infty[$ OR $y \geq -4$ OR $g \geq -4$ OR $[-4, \infty)$

[3 marks]

(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, \ p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t-1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

Markscheme

equating the two functions or equations ***(M1)***

$g(x) = j(x)$ OR $px^2 + (t-1)x - p = -x + 3p$

$px^2 + tx - 4p = 0$ ***(A1)***

attempt to find discriminant (do not accept only in quadratic formula) ***(M1)***

$\Delta = t^2 + 16p^2$ ***A1***

$\Delta = t^2 + 16p^2 > 0$, because $t^2 \geq 0$ and $p^2 > 0$, therefore the sum will be positive ***R1R1***

Note: Award ***R1*** for recognising that Δ is positive and ***R1*** for the reason.

There are two distinct points of intersection between the graphs of g and j. ***AG***

[6 marks]

21. [Maximum mark: 7] 22M.1.SL.TZ1.6

Consider $f(x) = 4\sin x + 2.5$ and
$g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a) Describe these two transformations. [2]

Markscheme

translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction ***A1***

translation (shift) by q upwards/positive vertical direction ***A1***

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

(b) The y-intercept of the graph of g is at $(0,\ r)$.

Given that $g(x) \geq 7$, find the smallest value of r. [5]

Markscheme

METHOD 1

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) *(M1)*

$-4+2.5+q\geq 7$

$q\geq 8.5$ (accept $q=8.5$) ***A1***

substituting $x=0$ and their q $(=8.5)$ to find r *(M1)*

$(r=)\ 4\sin\left(\frac{-3\pi}{2}\right)+2.5+8.5$

$4+2.5+8.5$ *(A1)*

smallest value of r is 15 ***A1***

METHOD 2

substituting $x=0$ to find an expression (for r) in terms of q *(M1)*

$(g(0)=r=)\ 4\sin\left(\frac{-3\pi}{2}\right)+2.5+q$

$(r=)\ 6.5+q$ ***A1***

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 *(M1)*

$-4+2.5+q\geq 7$

$-4+2.5+(r-6.5)\geq 7$ (accept $=$) *(A1)*

smallest value of r is 15 ***A1***

METHOD 3

$4\sin\left(x-\frac{3\pi}{2}\right)+2.5+q=4\cos x+2.5+q$ ***A1***

y-intercept of $4\cos x+2.5+q$ is a maximum *(M1)*

amplitude of $g(x)$ is 4 *(A1)*

attempt to find least maximum *(M1)*

$r = 2 \times 4 + 7$

smallest value of r is 15 ***A1***

[5 marks]

22. [Maximum mark: 5] 22M.1.SL.TZ2.5

Find the least positive value of x for which $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$. [5]

Markscheme

determines $\frac{\pi}{4}$ (or 45°) as the first quadrant (reference) angle ***(A1)***

attempts to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}$ ***(M1)***

Note: Award ***M1*** for attempting to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}, \frac{7\pi}{4}(, \ldots)$

$\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow x < 0$ and so $\frac{\pi}{4}$ is rejected ***(R1)***

$\frac{x}{2} + \frac{\pi}{3} = 2\pi - \frac{\pi}{4} \left(= \frac{7\pi}{4}\right)$ ***A1***

$x = \frac{17\pi}{6}$ (must be in radians) ***A1***

[5 marks]

23. [Maximum mark: 6] 22M.1.SL.TZ2.4

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1.$

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

Markscheme

$x = -1$ ***A1***

[1 mark]

(a.ii) Write down the equation of the horizontal asymptote. [1]

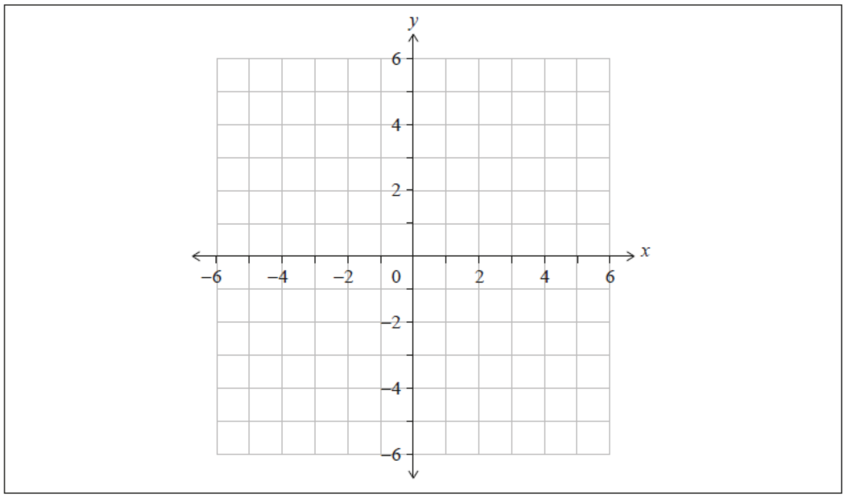
Markscheme

$y = 2$ ***A1***

[1 mark]

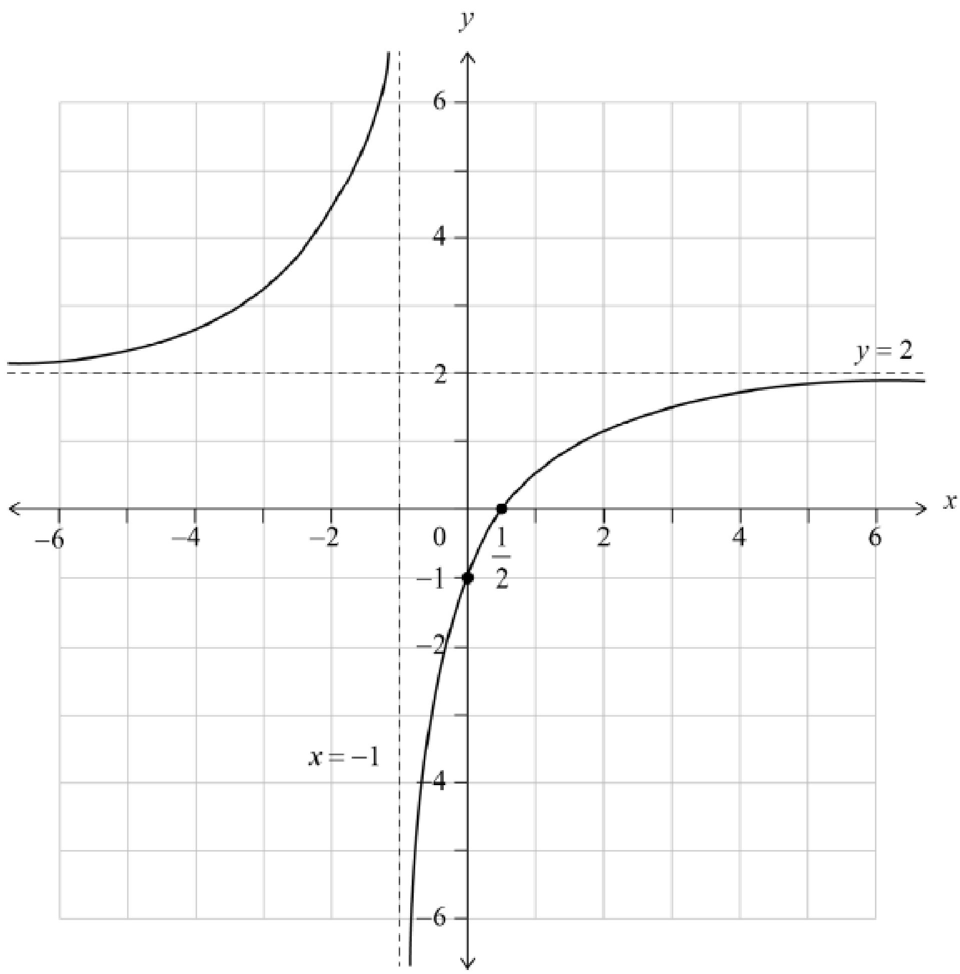
(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

Markscheme



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

Note: The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x=-1$ and $y=2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x=\frac{1}{2}$ and $y=-1$ ***A1A1***

[3 marks]

(c) Hence, solve the inequality $0<\frac{2x-1}{x+1}<2$. [1]

Markscheme

$x>\frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2},\ \infty\right)$ and $\left]\frac{1}{2},\infty\right[$.

[1 mark]