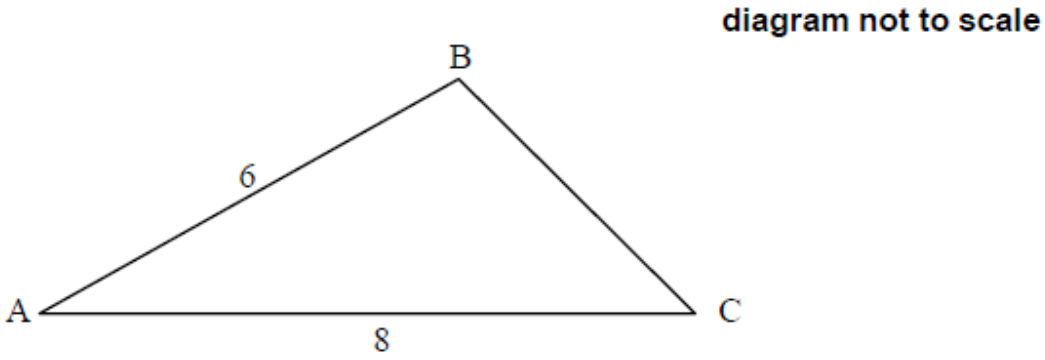
Revision (no GDC) [190 marks]

1. [Maximum mark: 5] SPM.1.SL.TZ0.1

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



(a) Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. [3]

(b) Find the area of triangle ABC. [2]

2. [Maximum mark: 5] SPM.1.SL.TZ0.5

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

3. [Maximum mark: 5] EXN.1.SL.TZ0.2

Solve the equation $2 \ln x = \ln 9 + 4$. Give your answer in the form $x = pe^q$ where $p, q \in \mathbb{Z}^+$. [5]

4. [Maximum mark: 6] EXN.1.SL.TZ0.4

The first three terms of an arithmetic sequence are u_1, $5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

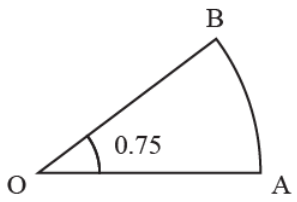
(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

5. [Maximum mark: 5] 24N.1.SL.TZ1.2

Points A and B lie on a circle with centre O and radius r cm, where $\mathrm{A\widehat{O}B} = 0.75$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $6\ \mathrm{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

6. [Maximum mark: 6] 24N.1.SL.TZ1.6

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$. [6]

7. [Maximum mark: 14] 24N.1.SL.TZ1.8

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(a.i) $f(2)$; [2]

(a.ii) $f\left(\frac{1}{8}\right)$. [1]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f\left(4x^2\right)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

8. [Maximum mark: 16] 24M.1.AHL.TZ2.10

Consider the arithmetic sequence $a,\ p,\ q \ldots$, where $a,\ p,\ q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence $a,\ s,\ t \ldots$, where $a,\ s,\ t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9,\ s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(d.i) arithmetic sequence; [2]

(d.ii) geometric sequence. [2]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

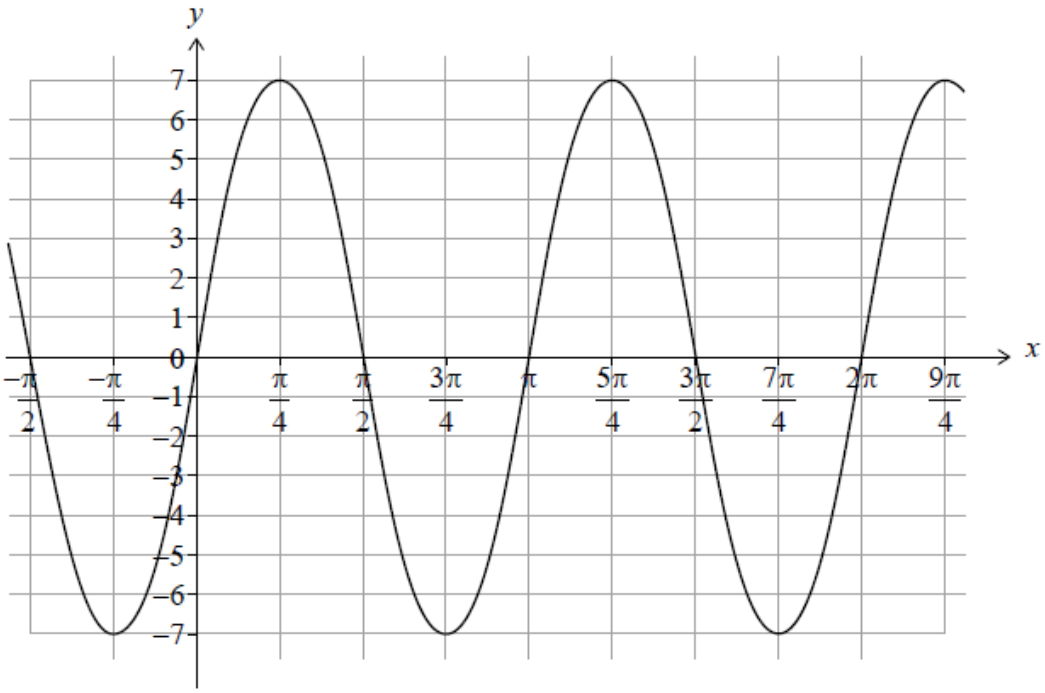
The first three terms of u_n are $u_1 = 9 + \ln 9,\ u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e.i) Find the common difference of the new sequence in terms of $\ln 3$. [3]

(e.ii) Show that $\sum_{i=1}^{10} = -90 - 25 \ln 3$. [3]

9. [Maximum mark: 7] 23N.1.SL.TZ1.1

Consider the function $f(x) = a\ \sin\ (bx)$ with $a,\ b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.



(a) Write down the value of a. [1]

(b.i) Write down the period of f. [1]

(b.ii) Hence, find the value of b. [2]

(c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

10. [Maximum mark: 5] 23N.1.SL.TZ1.2

Consider the functions $f(x) = x + 2$ and $g(x) = x^2 - k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(4) = 11$, find the possible values of k. [3]

11. [Maximum mark: 7] 23N.1.SL.TZ1.4

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_5 = 65$ and $S_6 = 96$.

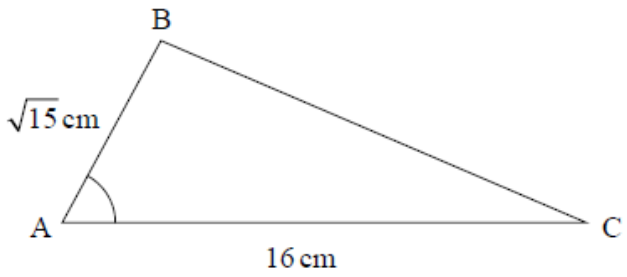
(a) Find the value of p and the value of q. [5]

(b) Find the value of u_6. [2]

12. [Maximum mark: 6] 23N.1.SL.TZ1.5

In the following triangle ABC, $\mathrm{AB} = \sqrt{15}\,\mathrm{cm}$, $\mathrm{AC} = 16\,\mathrm{cm}$ and $\cos \mathrm{B\hat{A}C} = \frac{1}{4}$.

diagram not to scale



Find the area of triangle ABC. [6]

13. [Maximum mark: 15] 23N.1.SL.TZ1.8

The functions f and g are defined by

$f(x) = \ln(2x - 7)$, where $x > \frac{7}{2}$

$g(x) = 2\ln x - \ln d$, where $x > 0,\ d \in \mathbb{R}^+$.

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$.

[1]

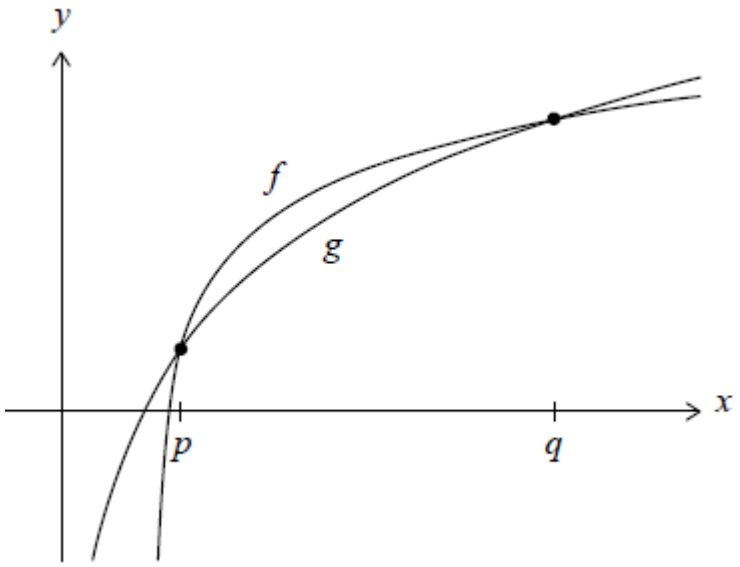
The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b.i) Show that, at the points of intersection,
$x^2 - 2dx + 7d = 0$. [4]

(b.ii) Hence, show that $d^2 - 7d > 0$. [3]

(b.iii) Find the range of possible values of d. [2]

The following diagram shows parts of the graph $y = f(x)$ and $y = g(x)$.



The graphs intersect at $x = p$ and $x = q$, where $p < q$.

(c) In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a,\ b \in \mathbb{Z}^+$. [5]

14. [Maximum mark: 7] 23M.1.SL.TZ1.2

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

(b.i) the vertical asymptote; [1]

(b.ii) the horizontal asymptote. [1]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

15. [Maximum mark: 14] 23M.1.SL.TZ1.8

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

(e) Given that $v_{99} < 0$, find v_5. [2]

16. [Maximum mark: 7] 23M.1.SL.TZ2.6

The functions f and g are defined for $x \in \mathbb{R}$ by

$f(x) = ax + b$, where $a, b \in \mathbb{Z}$

$g(x) = x^2 + x + 3$.

Find the two possible functions f such that
$(g \circ f)(x) = 4x^2 - 14x + 15$.

[7]

17. [Maximum mark: 5] 23M.1.SL.TZ2.3

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}, x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(a.i) the vertical asymptote; [1]

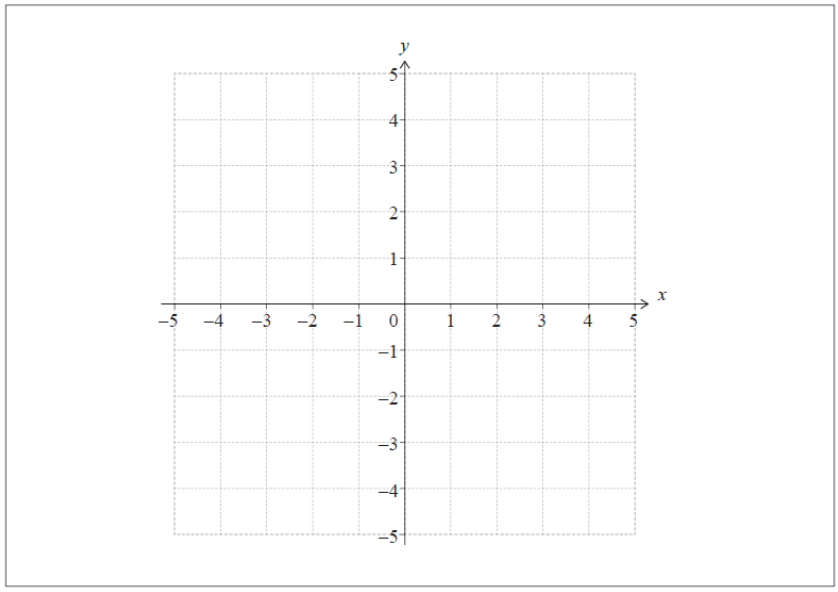
(a.ii) the horizontal asymptote. [1]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(b.i) the y-axis; [1]

(b.ii) the x-axis. [1]

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).

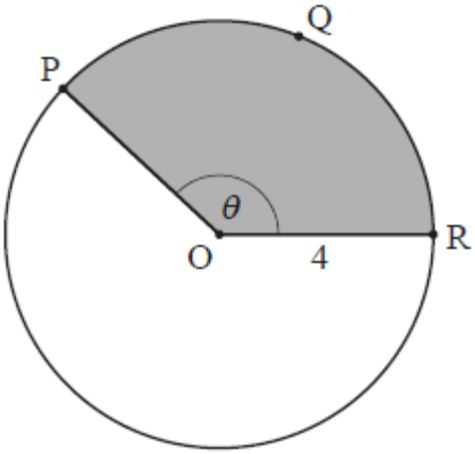


[1]

18. [Maximum mark: 6] 23M.1.SL.TZ2.1

The following diagram shows a circle with centre O and radius $4\ \mathrm{cm}$.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\mathrm{P\hat{O}R} = \theta$, where θ is measured in radians.

The length of arc PQR is $10\ \mathrm{cm}$.

(a) Find the perimeter of the shaded sector. [2]

(b) Find θ. [2]

(c) Find the area of the shaded sector. [2]

19. [Maximum mark: 15] 22N.1.SL.TZ0.8

Calculate the value of each of the following logarithms:

(a.i) $\log_2 \frac{1}{16}$. [2]

(a.ii) $\log_9 3$. [2]

(a.iii) $\log_{\sqrt{3}} 81$. [3]

It is given that $\log_{ab} a = 3$, where $a,\ b \in \mathbb{R}^+,\ ab \neq 1$.

(b.i) Show that $\log_{ab} b = -2$. [4]

(b.ii) Hence find the value of $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}$. [4]

20. [Maximum mark: 16] 22N.1.SL.TZ0.7

(a) The graph of a quadratic function f has its vertex at the point $(3,\ 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f\left(x\right) = a(x-h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t-1)x - p$ where $x \in \mathbb{R}$ and $p,\ t \in \mathbb{R},\ p \neq 0$.

In the case where $g(-3) = g(1) = 4$,

(b.i) find the value of p and the value of t. [4]

(b.ii) find the range of g. [3]

(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R},\ p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t-1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

21. [Maximum mark: 7] 22M.1.SL.TZ1.6

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a) Describe these two transformations. [2]

(b) The y-intercept of the graph of g is at $(0,\ r)$.

Given that $g(x) \geq 7$, find the smallest value of r. [5]

22. [Maximum mark: 5] 22M.1.SL.TZ2.5

Find the least positive value of x for which $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$. [5]

23. [Maximum mark: 6] 22M.1.SL.TZ2.4

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

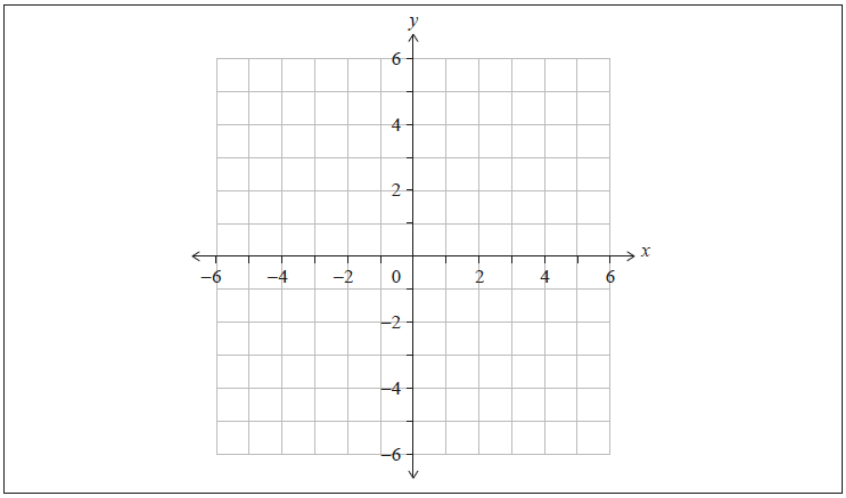
The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(a.i) Write down the equation of the vertical asymptote. [1]

(a.ii) Write down the equation of the horizontal asymptote. [1]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



[3]

(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. [1]