1. [5 *points*]

Let $f(x) = \dfrac{\arctan x^2}{\sin x}$. By writing:

$$\arctan(x^2) \equiv (\sin x)(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ...)$$

find the first two non-zero terms of the Maclaurin series for $f(x)$.

$$(x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - ...) \equiv (x - \frac{x^3}{6} + \frac{x^5}{120} - ...)(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ...)$$

Expanding and comparing coefficients we get:

$$a_0 = 0$$

$$a_1 = 1$$

$$a_2 - \frac{a_0}{6} = 0$$

$$a_3 - \frac{a_1}{6} = 0$$

Which gives:

$$f(x) = x + \frac{1}{6}x^3 + ...$$

2. [10 *points*]

(a) Find the first four non-zero terms of the Macluarin series for $\frac{1}{\sqrt{1+x}}$.

(b) Hence, or otherwise, find the first four non-zero terms of the Maclaurin series for $\frac{1}{\sqrt{1-x^2}}$.

(c) Hence, or otherwise, find the first three non-zero terms for the Maclaurin series for:

(i) $\frac{x}{\sqrt{(1-x^2)^3}}$

(ii) $\arcsin x$.

(a)

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots$$

(b)

$$\frac{1}{\sqrt{1-x^2}} = 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + \frac{5}{16}x^6 + \ldots$$

(c)

$$\frac{x}{\sqrt{(1-x^2)^3}} = \left(\frac{1}{\sqrt{1-x^2}}\right)' = x + \frac{3}{2}x^3 + \frac{15}{8}x^5 + \ldots$$

$$\arcsin x = \int \frac{1}{\sqrt{1-x^2}}dx = a_0 + x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \ldots$$

and since $\arcsin 0 = 0$ we have $a_0 = 0$, so:

$$\arcsin x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \ldots$$

3. [5 *points*]

Find the Maclaurin series up to x^4 term for e^{e^x-1}.

$$e^{e^x-1} = 1+(x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+...)+\frac{(x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+...)^2}{2}+\frac{(x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+...)^3}{6}+\frac{(x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+...)^4}{24}+...$$

this gives:

$$e^{e^x-1} = 1+x+x^2+\frac{5}{6}x^3+\frac{5}{8}x^4+...$$

Note that the function e^{e^x-1} is a generating function for Bell numbers - the number of partitions of a set. You may want to look into this and maybe investigate this as in your IA.