Binomial theorem [57 marks]

1. [Maximum mark: 4]

Find the coefficient of x^8 in the expansion of $(2x - 5)^{11}$. [4]

2. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by

$1 + \frac{9x}{2} + 15k^2x^2 + \ldots + k^nx^n$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of *n* and the value of *k*. [6]

3. [Maximum mark: 7]

The coefficient of x^6 in the expansion of $\left(ax^3 + b\right)^8$ is 448.

The coefficient of x^6 in the expansion of $\left(ax^3 + b\right)^{10}$ is 2880.

Find the value of a and the value of b, where a, $b > 0$. [7]

4. [Maximum mark: 6]

Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$.

Find the value of a. [6]

5. [Maximum mark: 5]

In the expansion of $(x + k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63.

Find the possible values of k. [5]

6. [Maximum mark: 5]

Consider the expansion of $\left(3 + x^2\right)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of x^4 is $20\,412$, find the value of n. [5]

7. [Maximum mark: 6]

Consider the expansion of $\left(3x^2 - \frac{k}{x}\right)^9$, where $k > 0$.

The coefficient of the term in x^6 is 6048. Find the value of k. [6]

8. [Maximum mark: 6]

Find the term independent of x in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2} - \frac{x}{2}\right)^9$. [6]

9. [Maximum mark: 7]

Consider the binomial expansion

$(x + 1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \ldots + 1$ where $x \neq 0$ and $a,\ b \in \mathbb{Z}^+$.

(a) Show that $b = 21$. [2]

(b) The third term in the expansion is the mean of the second term and the fourth term in the expansion.

Find the possible values of x. [5]

10. [Maximum mark: 5]

Consider the expansion of $\left(8x^3 - \frac{1}{2x}\right)^n$ where $n \in \mathbb{Z}^+$. Determine all possible values of n for which the expansion has a non-zero constant term. [5]