TEST 2

- The test consists of two sections. In section A calculators are **not allowed**. Graphic display calculator is required for section B.

- Unless otherwise stated in the question, all numerical answers should be given exactly, **fully simplified**, or correct to three significant figures.

- The maximum mark for this test is [**36 + 36 marks**].

- Time allowed is **90 minutes**.

- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.

SECTION A

1. [10 *points*]

Solve following equations/system of equations. Give your answers exactly, with rational denominators (if applicable).

(a) $4x + 3\sqrt{2} = x\sqrt{2} - 4$ [2]

(b) $2x^2 - 20 - 2x = x^2 - 5$ [2]

(c) $\dfrac{9^{x+2}}{\sqrt{27}} = (\sqrt[3]{81})^{2x+3}$ [2]

(d) $\begin{cases} 2x + 5y = 29 \\ 4x - 2y = -2 \end{cases}$ [2]

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(e) $\dfrac{|x-2|}{2} - \dfrac{|4-2x|}{3} = 1$ [2]

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2. [10 *points*]

Solve the following inequalities and systems of inequalities. Give your answers using interval notation.

(a) $2x - \dfrac{x-1}{4} > \dfrac{x}{2}$ [2]

(b) $\sqrt{x^2 + 6x + 9} + 2 \leqslant 8$ [2]

(c) $2x^2 - 9x + 8 \geqslant -1$ [2]

(d) $\begin{cases} x^2 \leqslant 81 \\ x^2 + 6x \geqslant -5 \end{cases}$ [4]

3. [4 *points*]

Simplify the following algebraic fraction. State all necessary assumptions.

$$\frac{x^4 - 25x^2 + 144}{x^3 - 4x^2 - 9x + 36}$$

4. [6 *points*]

The first three terms of an arithmetic sequence are:

$$10k + 1, \quad 2k, \quad 4k^2 - 5$$

(a) Find the possible values of k. [2]

(b) For each value of k, write down the common difference of the sequence. [2]

(c) For the integer value of k find the explicit (general) formula for the n-th term of the sequence. [2]

5. [6 *points*]

Consider the equation:

$$x^2 - 2m(x-1) - 1 = 0$$

(a) Find all possible values of parameter m for which the equation has two distinct real solutions. [3]

(b) Find value of m for which ratio of product and sum of solutions is equal to 4. [3]

SECTION B

1. [8 *points*]

Solve the following equations and systems of equations. Give your answers exactly or correct to 3 significant figures.

(a) $x\sqrt{3}+5=-x\sqrt{5}+2\sqrt{5}$ [2]

(b) $\frac{2}{3}x^2+3(x+3)=21(x-2)$ [2]

(c) $4\times(\sqrt[3]{5})^{6x+3}+2=500$ [2]

(d) $\begin{cases}4x-5y=10\\ \frac{1}{2}x-6y=-5\end{cases}$ [2]

2. [10 *points*]

Consider the following sequences:

$$a : 4,\ 12,\ 36,\ 82,\ 156,\ ...$$

$$b : 4,\ 12,\ 36,\ 108,\ 324,\ ...$$

(a) Decide if the sequences are arithmetic (linear), quadratic, cubic, geometric (exponential) or none of the above. Justify your answer. [2]

(b) Find the general (explicit) or recursive formula for each sequence. [4]

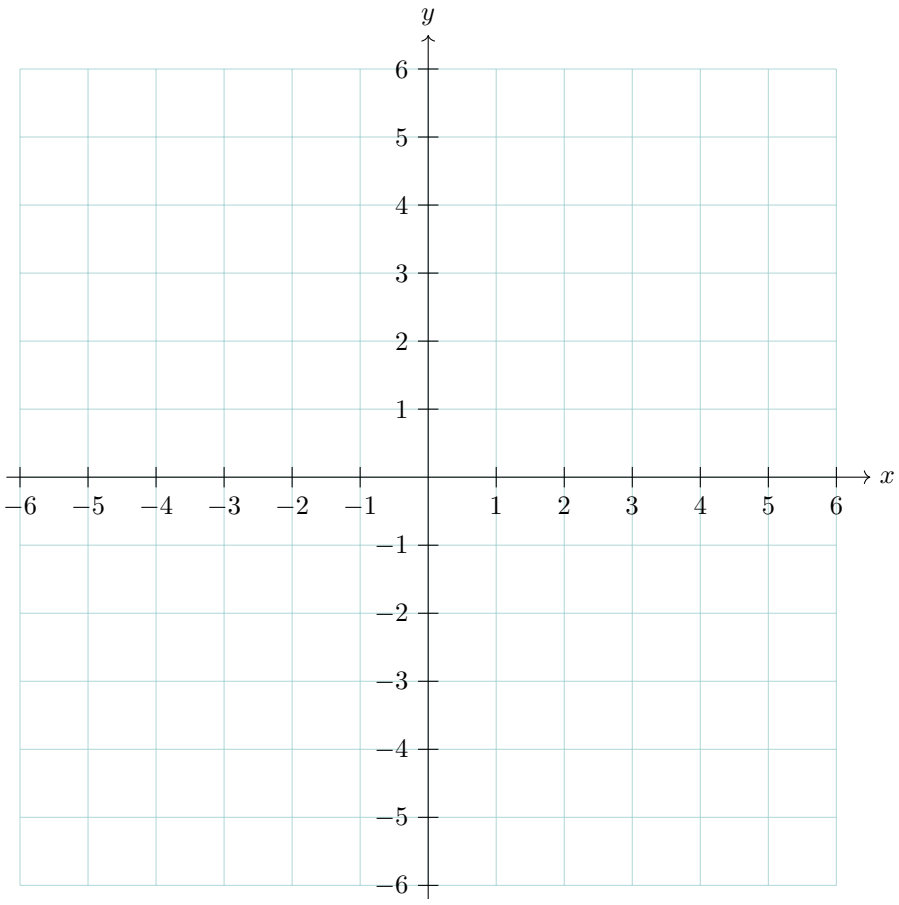
(c) Find the number of terms of each sequence which are smaller than 2000. [4]

3. [6 *points*]

Consider the following system of inequalities:

$$\begin{cases} y < x \\ y < 1 \\ x < 0 \end{cases}$$

(a) Represent the set of points satisfying the above inequalities by shading an appropriate region on the diagram below. [3]



(b) Decide if the following points satisfy the above system of inequalities: [3]

(i) $(-1, -1)$ (ii) $(2, -5)$ (iii) $(-1, -2)$

4. [4 *points*]

Consider the following intervals:

$$A = [2 - \sqrt{2}, \infty[\qquad B = [-3, 3[\qquad C =]-\infty, 4] - \{-3\}$$

(a) Write down: [3]

(i) $B \cap C$

(ii) $A \cap B \cap \mathbb{Z}$

(iii) $C' \cup B$

(b) Is it true that $\sqrt{2} - 2 \in B - A$. Justify your answer. [1]

5. [8 *points*]

Consider the following quadratic sequence:

$$4,\ 6,\ 4,\ -2,\ -12,\ ...$$

(a) Write down the next (sixth) term of this sequence. [1]

(b) Find the general formula of this sequence in the form: [2]

$$u_n = an^2 + bn + c$$

(c) The last term of this sequence is equal to -506. Calculate how many terms are in this sequence. [2]

(d) What is the value of the first term less than -254 [2]

(e) Check, if number -122 is a member of this sequence. Justify your answer. [1]